Cognitive and Noncognitive Factors in Educational Production

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Abstract

This study examines how cognitive and noncognitive abilities jointly influence academic performance in Maths and English among Irish secondary students, with a focus on gender differences. Using data from the Growing Up in Ireland longitudinal study, I apply linear and translog production functions to model these relationships. The results reveal that cognitive abilities are the strongest predictors of academic performance for all students, with a slightly stronger correlation observed for male students. Noncognitive factors, particularly Focused Behavior and Conscientiousness, also contribute significantly, especially for female students in Maths. The interaction between cognitive and noncognitive factors varies by subject: in Maths, they tend to complement each other, while in English, one can more easily compensate for the other. However, there is a limit to how much noncognitive abilities can make up for cognitive skills, with behavioral skills offering more flexibility than personality traits. The study also uncovers diminishing returns as students develop their abilities, particularly in English. These findings suggest that educational strategies targeting both cognitive and noncognitive development, tailored to gender and subject-specific needs, may be more effective in addressing academic achievement gaps. The results also highlight the importance of considering whether noncognitive measures represent changeable behaviors or more stable personality traits when designing educational interventions.

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Contents

I. Introduction

Human capital development plays a central role in economic growth and individual labor market outcomes. While existing literature has extensively explored the roles of cognitive and noncognitive skills in academic achievement, important gaps remain in our understanding of their joint effects and potential for substitution or complementarity. Specifically, previous studies have often treated cognitive and noncognitive skills as independent factors, overlooking the intricate relationship between them in shaping educational outcomes. In addition, there is a lack of research examining how these connections might vary across different academic subjects and genders.

In this study, I address these gaps by examining the interaction between cognitive and noncognitive skills in predicting academic performance in Maths and English among secondary-level students in Ireland. The Irish educational context provides a unique backdrop for this research, characterized by: a centralized curriculum and standardized national examinations (e.g., the Junior Certificate), allowing for consistent measurement of academic achievement across the country; recent emphasis on ”wellbeing” in the curriculum (Lawlor, 2019; National Council for Curriculum and Assessment, 2021), highlighting the growing recognition of noncognitive skills in Irish education policy; and a persistent gender gap in STEM subjects (McNally, 2020), making the exploration of gender differences in skill utilization particularly relevant.

Specifically, I aim to answer the following research questions:

1. To what extent do cognitive and noncognitive skills contribute to academic achievement in Maths and English?

2. How do these contributions differ between genders?

3. What is the nature of the interaction between cognitive and noncognitive skills in predicting academic performance?

Using data from the Growing Up in Ireland longitudinal study, I employ regression analysis and estimate a translog production function to model the relationship between cognitive skills, noncognitive skills, and academic achievement. This approach allows me to quantify non-linear links and gender-specific effects, providing a more nuanced understanding of the educational production process.

My findings reveal several important insights:

- Cognition consistently emerges as the strongest predictor of academic performance, with boys showing slightly higher cognitive output elasticities than girls in both subjects.

- Noncognitive factors, particularly behavioral skills (measured by SDQ), have smaller but significant effects, with stronger impacts for girls, especially in Maths.

- Both cognitive and noncognitive factors have a more pronounced influence on Maths performance compared to English, suggesting subject-specific dynamics in skill utilization.

- All models exhibit decreasing returns to scale, with higher returns for Maths than for English.

- The Elasticity of Substitution varies across models and subsamples, indicating complementarity between cognitive and noncognitive skills in Maths and substitutability in English.

- The Marginal Rate of Technical Substitution suggests that while noncognitive skills can compensate for cognitive skills to some extent, there are limitations to this substitution.
- The translog model reveals a substitutive relationship between cognitive and noncognitive skills not apparent in the simpler Cobb-Douglas specification.

These insights have important implications for educational policy, suggesting that interventions aimed at developing both cognitive and noncognitive skills may be more effective than those focusing on either in isolation. Furthermore, my findings highlight the need for tailored approaches that consider gender differences and subject-specific dynamics in skill development.

While extensive research has been conducted on cognitive and noncognitive skills in education (Cunha & Heckman, 2008; Heckman & Kautz, 2012a), and some studies have examined gender differences in academic performance (Hyde, 2016; Niederle & Vesterlund, 2010), my study uniquely combines these elements within the Irish educational context. By employing both linear and translog production functions, I was able to extend the methodological approaches typically used in educational production function research (Todd & Wolpin, 2003). My focus on subject-specific interactions between cognitive and noncognitive skills, particularly in Maths and English, builds upon but differs from previous work that has often treated these skills more uniformly across subjects (Balart et al., 2018). Furthermore, by situating this analysis within the Irish secondary education system, this study contributes to the understanding of these dynamics in a specific national context, building upon previous Irish educational research (Smyth et al., 2015; Sofroniou et al., 2000). The use of both the SDQ (Goodman, 1997) and TIPI (Gosling et al., 2003) provides a comprehensive measure of noncognitive skills while also allowing for a extensive analysis of their role in academic achievement. By combining these different approaches, my study reveals new observations about how various factors work together to affect academic performance.

These findings could help shape more effective, targeted educational strategies, for example the implementation of personalized learning plans; the creation of programs which focus on developing students' abilities to understand and manage emotions, set goals, show empathy, establish relationships, and make responsible decisions; the establishment of workshops and activities designed to enhance specific skills, such as teamwork, communication, and problem-solving that can be integrated into school curricula or after-school programs, as proposed by Durlak et al. (2011).

The remainder of this paper is structured as follows: Section I provides an overall review of the literature on cognitive and noncognitive skills in education, with a focus on gender differences. Section II describes the data used in this study, including details on the Growing Up in Ireland longitudinal study, the chosen instruments, and its limitations. Section III presents the theoretical framework, a model on the linear production function and its estimation. In section IV I extend the model to allow for nonlinearity by estimating a translog production function. Section V then concludes with a broader discussion of the overall results, a summary of the key contributions of this study, the possible limitations, and directions for future research.

A. The multidimensional nature of education and skills

Education is, in its nature, multidimensional, occurring in a feedback loop across time and space and involving numerous agents and institutions. Given the scarcity of resources such as money and labour, it is essential to allocate them wisely to achieve the most effective outcomes. Traditionally, the effectiveness of educational resources has been measured through completed levels and years of education (quantity, educational attainment) and test scores (quality, educational achievement). More properly qualified and skillful students lead better lives and participate more actively in civic duties and the labour market (Oreopoulos & Salvanes, 2011).

However, a growing body of empirical research suggests that this view may be too narrow. Kautz et al. (2014) note that noncognitive skills rival IQ in predicting educational attainment, labour market success, health, and criminality. This shift in perspective calls for a deeper understanding of what factors contribute to educational outcomes and how we can measure them. The literature on the returns to education, both private and social, is vast and almost unanimous on the importance of improving educational outcomes for students.

From an economics perspective, a skill is a form of human capital that increases productivity, with its value defined by the market. Education is perceived as an essential investment in skills development (Zhou, 2017). The literature typically divides skills into two categories: cognitive and noncognitive skills, a term first coined by James Heckman.

Levin (2012) argues for a broader perspective on educational outcomes, emphasizing that success in life depends on more than just test scores. This multidimensional view of education aligns with the growing recognition of noncognitive skills' importance in both academic and life outcomes.

B. Cognitive and noncognitive skills in academic achievement

Cognitive skills, often proxied by test scores, have long been considered the primary determinant of academic success. Heckman et al. (2006) provide evidence that cognitive skills are strong predictors of educational attainment and labour market success. However, it is important to note that test scores do not simply reflect cognitive ability. Brunello and Schlotter (2011) suggest that high cognitive test scores likely result not only from high cognitive skills but also from high motivation and adequate personality traits, which can be considered noncognitive skills.

Noncognitive skills enable people, fostering social inclusion and promoting economic and social mobility (Kautz et al., 2014). Bowles and Gintis (2002) found that perseverance, dependability, and consistency are some of the most important predictors of grades in school. Almlund et al. (2011) provide a comprehensive review of how personality traits influence educational outcomes, finding that conscientiousness, in particular, is a strong predictor of academic performance across various measures and educational levels.

The interaction between cognitive and noncognitive skills is nuanced. Borghans et al. (2008) state that a link between noncognitive skills and test scores can exist for two reasons: when there are sufficient rewards involved, people with favorable behavioural or labour-market outcomes might have an attitude to put in effort, and when rewards are not necessary, people who are motivated to perform well and who have a positive attitude toward work might be more inclined to do their best at tests.

Duckworth and Seligman (2006), in a study about the difference in test scores between girls and boys, concluded that because girls had better final grades than boys, even after controlling for measured IQ, they were significantly better at exercising self-discipline during the academic year. Balart et al. (2018) used the performance decline in PISA test scores as a measure of noncognitive skills. They found that both the starting performance (a measure of cognitive skills) and the performance decline were positively and significantly associated with economic growth. When controlling for the performance decline, the estimated effect of cognitive skills on economic growth was reduced by approximately 40 percent. This highlights the importance of considering both cognitive and noncognitive skills in research on economic growth and education.

Lindqvist and Vestman (2011) provide evidence on the relative importance of cognitive and noncognitive skills in predicting labour market outcomes. While their focus is on labour market success, their findings have implications for understanding how these skills interact in educational settings. They find that noncognitive skills are particularly important for individuals at the lower end of the earnings distribution, suggesting a potential compensatory effect.

C. Gender differences and skill development

Gender differences in academic performance have been a significant area of research. As mentioned earlier, Duckworth and Seligman (2006) found that girls had better final grades than boys, even after controlling for measured IQ, attributing this to girls' better self-discipline. Bertrand and Pan (2013) examined gender differences in noncognitive skills, focusing specifically on disruptive behaviour. They find that boys are more susceptible to developing behavioural problems, especially in disadvantaged environments, which can significantly impact their academic performance.

Skill development is a dynamic process in which the early years lay the foundation for successful investment in later years (Kautz et al., 2014). The work of Cunha and Heckman (2008) has been instrumental in formalizing the role of noncognitive skills in skill formation. They propose a model that incorporates both cognitive and noncognitive skills, highlighting how these skills interact and evolve over time. This model has been influential in shaping our understanding of skill development and its impact on educational outcomes.

Both cognitive and noncognitive skills have different levels of malleability depending on a child's developmental stage; they can change with age and with instruction. Cognitive and noncognitive skills are highly malleable in the early years of a child's life, while noncognitive skills are more malleable than cognitive skills later on, during adolescence (Kautz et al., 2014).

D. Challenges in defining and measuring noncognitive skills

Despite their importance, noncognitive skills and abilities, unlike cognition, are challenging to define and measure. Suárez Pandiello et al. (2016) attest that social groups and public authorities ignore noncognitive abilities because of the lack of objective evaluation metrics and the difficulty in establishing standard definitions for the relevant social values.

Humphries and Kosse (2017) note that the definition of noncognitive skills varies widely across fields such as Sociology, Psychology, and Economics and within fields of study. Labour economists see noncognitive skills as a second dimension of individual heterogeneity (next to cognitive skills); Education economists broadly categorize those as skills that are not captured by standardized tests (soft skills), and that can be measured by observing behaviour. behavioural economists are divided into two groups: one that sees noncognitive skills as a super-ordinate concept summarizing various specific concepts (i.e., economic preferences such as time and risk preferences), and the other that views them as personality measures (such as the Big Five). This divisiveness is challenging when comparing outcomes due to the different measurement instruments used.

Currently, there is no systematic global measure of noncognitive skills. However, fortunately, the field has expanded enough, and a wide variety of instruments aimed at assessing these skills and abilities have been created. Using measured behaviours to capture noncognitive skills, for example, is a promising, empirically practical approach, according to Kautz et al. (2014).

Personality traits represent relatively persistent dimensions of the overall personality, and some play an important role in increasing productivity-enhancing skills. More broadly, economists often use the term noncognitive skills to account for traits specifically related to human capital outcomes (such as educational and labour market achievements), and in Psychology, personality traits are measured using psychometric constructs (Thiel & Thomsen, 2013). Therefore, economists and other social scientists can adapt such constructs to their respective fields of study to measure noncognitive skills.

II. Data

A. Growing Up in Ireland

The data used in the analysis come from the second and third waves of the Child Cohort ('98) of the Growing Up in Ireland (GUI) survey. The GUI is a national longitudinal study of children and young people that has been running since 2006. The study followed the progress of two groups of children: 8,568 9-year-olds (Cohort' 98), representing approximately 14% of all 9-year-olds in Ireland, and 10,000 9-month-olds (Cohort' 08), for the last fifteen years. Subsequent waves of the '98 cohort saw some drop-off in participation: 7,525 children (87.9%) in the second wave (2011-2012), 6,216 young adults (72.5%) in the third wave (2015-2016), and 5,190 young adults (60%) in the fourth wave (2018-2019). The survey stands out for its large, nationally representative sample and longitudinal nature. The first cohort sample was selected from clustering at the school level, and the second cohort was sampled randomly from the Child Benefit records. The members of Cohort '98 are now 25-26 years old.

Event	**Date**	**Age (in years)**	**Variables of interest**
Study-child is born	Nov/97 - Oct/98	0	
Wave 2 data collection	Aug/11 - Mar/12	13	Independent variables: Cognition composite, SDQ and TIPI scales, controls
Study-child sits the Junior Cert	Jun/13 - Jun/14	15-16	
Wave 3 data collection	Apr/15 - Aug/16	17-18	Dependent variables: Junior Cert scores in Maths and English

Table 1: Timeline of Events - Growing Up in Ireland '98 Cohort

A timeline of the data used in this study is presented in Table 1. In Wave 2, the study children had their verbal reasoning and numerical abilities tested using the Drumcondra Verbal Reasoning, the Numerical Ability tests, and the Matrices British Ability Scale (one of the leading standardized batteries in the UK for assessing a child's cognitive ability and educational achievement)[1]. These measures were combined through principal component analysis, yielding a single component representing cognitive ability, where higher scores indicate more remarkable ability. I use this composite throughout the study as a measure of cognition. The noncognitive variables used in this study were also collected in Wave 2, along with the control variables such

[1]These tests compromise different cognitive abilities: verbal fluency, vocabulary comprehension, and numerical knowledge. The verbal fluency test encompassed two aspects: the FAS score, measuring the number of words generated beginning with F, A, or S in one minute, and the Animal Naming score, gauging the number of animal species named in one minute. The vocabulary test consisted of 20 items, each followed by a list of five words, requiring the selection of the word most closely related in meaning. The numeracy test evaluated performance in basic arithmetic through three mathematical calculations.

as socioeconomic indicators and school characteristics. In wave 3, they were asked about Junior and Leaving Cert (if they already sat it) results and asked for permission to link to the Central Admissions Office database in the future (if they still need to sit it).

Academic achievement at the third wave was assessed via the Junior Certificate Examination, a national exam taken by most Irish children around ages 15-16. Mandatory subjects are Irish, English, Maths, and History, and students can choose up to 10 subjects (with at least four mandatory plus two optional) in the areas of Arts and Humanities, Modern Languages, Sciences, and Applied Sciences. Before 2017 (when the survey took place), grades were given on a scale of A to F across different levels of the exam (Higher, Ordinary, Foundation). The Junior Certificate Examination in Ireland marks the end of three years of studying various subjects. It typically spans two to three weeks of individual subject exams at the end of the school year in June, and a student cannot fail the examination. Regardless of their examination results, all students progress to the next year of education if they wish to do so. The time frame between the Junior Certificate (ages 15-16) and the age range of the GUI wave 3 participants (16-18) was relatively close. Because the Junior Cert syllabus and exam content are predetermined three years in advance, achieving success reflects the culmination of a structured curriculum and learning process. Given this foresight, one would anticipate that specific noncognitive skills are pivotal in shaping outcomes. These skills may include effective planning, adept time management, the ability to prioritize long-term goals over immediate gratification (such as opting to study for an exam well in advance rather than indulging in leisure activities), proficient organization and upkeep of study materials, and judicious allocation of time across a diverse array of academic subjects.

B. Variables

The model I employ at first is a multivariate multiple regression model (multivariate because of two dependent variables - Maths and English scores - and multiple because of multiple independent variables) as a form of linear production function.

1. Dependent variables

The dependent variables are Junior Cert scores in Maths and English, representing academic achievement. For the analysis, I used the Junior Certificate Overall Performance Scale (OPS), which converts letter grades from different exam levels to a standardized 12-point numerical scale. This scale has been validated in previous research (Sofroniou et al., 2000) and provides a comprehensive measure that accounts for both grade and exam level, allowing for more nuanced statistical analysis of academic achievement across subjects and students.

2. Independent variables

The independent variables consist of two sets of measures. The first set comprises cognitive abilities, assessed through naming ability, maths ability, and vocabulary ability. The second set includes noncognitive abilities and skills, measured using two different instruments: the Strengths and Difficulties Questionnaire (SDQ), which captures behavioural and emotional characteristics (also referred as psychosocial attributes), and the Ten Item Personality Inventory (TIPI), which assesses the five-factor model of personality.

Table 2: Descriptive Statistics - Main Variables

Variable	Mean	Std. Dev.	Min	Max	N
Dependent variables					
Maths points (Junior Cert)	9.60	1.74	2.00	12.00	5631
English points (Junior Cert)	10.15	1.34	5.00	12.00	5631
Independent variables: Cognition					
Drumcondra Verbal Reasoning (% of correct answers)	64.89	21.92	0.00	100.00	5631
Drumcondra Numerical Ability (% of correct answers)	55.05	22.53	0.00	100.00	5631
Matrices (BSA)	116.68	18.03	10.00	161.00	5631
Cognitive ability 1	0.14	1.33	-4.25	3.32	5631
Cognitive ability 2	100.00	15.00	36.25	136.40	5631
Independent variables: Noncognition (SDQ scale)					
Emotional resilience	8.29	1.87	0.00	10.00	5631
Good conduct	8.97	1.31	0.00	10.00	5631
Focused behaviour	7.56	2.26	0.00	10.00	5631
Positive peer relationships	8.96	1.41	0.00	10.00	5631
Independent variables: Noncognition (TIPI scale)					
Agreeable	5.01	1.95	0.50	7.00	5631
Conscientious	4.33	2.07	0.50	7.00	5631
Emotional stability	4.40	1.99	0.50	7.00	5631
Extravert	3.98	1.98	0.50	7.00	5629
Openness	4.73	1.83	0.50	7.00	5627
Controls (SES characteristics)					
Gender (Male = 1)	0.49	0.50	0.00	1.00	5468
Primary caregiver education level	3.97	1.24	1.00	6.00	5631
Secondary caregiver education level	3.86	1.36	1.00	6.00	4440
Income quintile (equivalized)	3.33	1.39	1.00	5.00	5241
Controls (School characteristics, binary)					
DEIS (Delivering Equality of Opportunity In Schools)	0.12	0.33	0.00	1.00	5452
Fee-paying	0.10	0.30	0.00	1.00	5452
Mixed-school	0.54	0.50	0.00	1.00	5317

Note: TIPI scale scores on a 1-7 scale in intervals of 0.5, and the original SDQ scales, ranging from 0 to 10, have been inverted (higher scores typically indicate more problems in the original SDQ scale). ”Cognitive ability 1” was used in the first part of the production function estimation and was standardized to have mean = 0 and standard deviation = 1. ”Cognitive ability 2” is to be used in the second part of the analysis as a measure of cognition in non-linear production function estimation, with a mean of 100 and standard deviation = 15 as is standard in the literature. Education levels are coded from 1 (Primary or less) to 6 (Postgraduate/Higher degree) in the Growing Up in Ireland caregiver questionnaire. The mean values for both primary (3.97) and secondary (3.86) caregivers indicate an average education level between Leaving Certificate and Diploma/Certificate, suggesting a higher proportion of educated caregivers in the sample. Income is reported in quintiles, where 1 represents the lowest 20% and 5 the highest 20% of incomes. The mean of 3.33 suggests that the sample is slightly skewed towards higher income levels, with families on average being just above the median income quintile. The sample includes 12% DEIS schools (schools in disadvantaged areas), 10% fee-paying schools, and 54% mixed-gender schools. This suggests a diverse range of school types, with a notably high proportion of fee-paying schools and a relatively low proportion of DEIS schools.

3. Controls

In addition, I include two vectors of control variables. The first vector encompasses socioeconomic status characteristics, including gender, parental education (considering both primary and secondary caregivers' education levels), and income quantile (equivalized). These SES variables are included to control for well-documented effects of family background on educational outcomes (Sirin, 2005). Gender is included to account for potential differences in subject-specific performance (Hyde & Linn, 1988; Hyde et al., 1990), while parental education and income are key indicators of family resources and educational support (Davis-Kean, 2005).

The second vector accounts for school characteristics, incorporating indicators for mixed schools (opposite to single-sex schools, which are underrepresented in the sample), DEIS (Delivering Equality of Opportunity in Schools) schools, and fee-paying schools. School-level variables are included to account for institutional factors that may influence academic performance. The inclusion of indicators for mixed schools addresses potential differences in educational environments (Pahlke et al., 2014). DEIS school status is included to control for the effects of targeted educational interventions in disadvantaged areas (Smyth et al., 2015). Fee-paying school status is included to account for potential resource differences between public and private institutions (OECD, 2012).

C. Intruments

1. Strengths and Difficulties Questionnaire (SDQ)

The Strength and Difficulties Questionnaire (SDQ) measures two distinct dimensions of noncognitive skills: behavioural skills and emotional skills. Twenty items of the SDQ comprise a total scale made up of four sub-scales, each containing five items. These sub-scales tap into emotional symptoms (e.g. often unhappy, downhearted, or tearful); conduct problems (e.g. often fights with other children or bullies them); Hyperactivity/Inattention (e.g. restless, overactive, cannot stay still for long); and Peer-relationship problems (e.g. picked on or bullied by other children). Scores on each sub-scale can range from 0 to 10, where 10 indicates a high degree of difficulty and 0 the absence of any problems in the relevant domain.

I inverted the scales so that 10 is better and 0 is worse, which led me to rename the measures to maintain clarity and consistency across the study. For example, a positive coefficient for Emotional Resilience (previously Emotional Symptoms) would indicate that higher emotional stability is associated with better academic outcomes. The same rationale was applied to the other variables: Conduct Problems became Good Conduct, Hyperactivity/Inattention became Focused Behaviour, and Peer-relationship Problems became Positive Peer Relationships. The SDQ was completed by both the child's primary caregiver and teacher in Wave 1, and by the child's primary caregiver in Wave 2.

2. Ten Item Personality Inventory (TIPI)

One of the dimensions where noncognition manifests itself (others being through behavioural problems, social skills, communication, self-esteem, persistence, locus of control, empathy, and impulsivity), the study-child was assessed utilizing the Ten Item Personality Inventory (TIPI), a brief instrument designed to assess the five-factor model (FFM) personality dimensions. Primary caregiver (PCG, usually the mother) and Secondary caregiver (SCG, usually the father) completed the scale regarding the study-child in wave 3 (PCG completed in waves 2 and 3). In wave 3, the study child also filled out the scale, offering an external and self-assessed measure of the study child's personality and ensuring consistency. This scale comprises ten items encompassing five personality facets: Openness to Experience, Agreeableness, Conscientiousness, Extraversion, and Neuroticism (Emotional stability). Each of the ten items was evaluated on a seven-point scale, from strongly disagree to strongly agree. Each dimension of personality included two statements with two descriptors each. The scores for each measure were derived by summing up both responses and dividing by two according to common practice in the literature. This was done by the GUI researchers and the final score for each item can be found in the GUI files. More details can be found in the Appendix.

D. Limitations

There are certain limitations to this analysis. I chose to work with the cross-sectional part of the panel data, which limits the ability to infer causality. There may also be omitted variable bias, as other factors not included in the model could influence academic performance (like residing in a peaceful environment, or just waking up well-rested in the Junior Cert days).

Furthermore, the parent-reported nature of some measures is potentially a source of measurement error. It is important to note that while regressing test scores on other test scores can sometimes lead to issues of regression to the mean, my study design mitigates this concern. The cognitive variables were collected two years before the study children sat the Junior Cert, allowing them to function as true predictors rather than concurrent measures. This temporal separation between the collection of cognitive and noncognitive measures (Wave 2) and the assessment of academic achievement (Wave 3) strengthens the predictive power of my analysis because it allows us to examine how earlier cognitive and noncognitive traits influence later academic outcomes, reducing concerns about reverse causality.

Regarding the noncognitive measures, while the TIPI and SDQ are widely used and validated instruments, they have inherent limitations. The TIPI's brevity, while efficient, may limit its ability to capture nuanced personality traits. The SDQ, although comprehensive, may be subject to reporter bias. Both measures rely on self or parent reports, which can introduce subjective biases. However, their established validity in Psychology and Sociology research and their efficiency in large-scale studies provide a strong foundation for their use in this analysis, balancing practical considerations with scientific rigor.

It is worth noting that the Growing Up in Ireland is a panel-data survey, and in Wave 3, the TIPI scores for the study children were also collected from the children's perspective and the secondary caregivers' perspective, providing three measures of noncognition from different viewpoints. These measures correlate well. By leveraging this survey's strength, I was able to minimize individual reporting biases and enhance construct validity through convergence of scores across different reporters. This approach provides a richer, more comprehensive view of children's noncognitive traits while increasing overall measurement reliability. The comparability of results across these different informants strengthens my confidence in the validity and robustness of our noncognitive measures, particularly the TIPI scores.

III. How do cognitive and noncognitive skills contribute to academic achievement? A linear approach

A. Theoretical Framework

In the fields of Economics of Education, Psychology, and Sociology, understanding the factors that contribute to academic achievement is central for developing effective policies and interventions. While cognitive abilities have traditionally been the primary focus when examining determinants of educational outcomes, recent interdisciplinary research has highlighted the significant role of noncognitive skills in shaping academic performance and long-term success.

To capture a detailed relationship between cognition and noncognition, I propose creating a series of educational production functions that incorporate both as inputs, which I term a grade production function (or performance function). This conceptual tool models the links between inputs and educational outcomes, with the primary focus on:

- Assessing the relative returns to cognitive and noncognitive measures, and
- Investigating potential interactions between these two types of abilities, specifically whether they act as complements or substitutes in producing educational outcomes.

B. Model Specification

I employ a linear form of the production function to estimate the effects of cognitive and noncognitive abilities on academic performance, while also capturing potential interactions between these factors. This model:

1. Estimates the direct effects of cognitive abilities and various noncognitive measures on academic points in Junior Certificate subjects;
2. Captures potential complementarities or substitutabilities between cognitive and noncognitive abilities through interaction terms;
3. Controls for relevant socioeconomic and school characteristics to isolate the effects of interest.

The linear production function is specified as follows:

$$\begin{aligned}\text{Points\_JC}_{i,w,l} = \beta_0 + \beta_C \cdot \text{Cognition}_{i,w} + \sum_{j=1}^{J} \beta_{Nj} \cdot \text{NonCognition}_{i,w,k,j} + \\ \sum_{j=1}^{J} \gamma_j \cdot (\text{Cognition}_{i,w} \cdot \text{NonCognition}_{i,w,k,j}) + \delta' \cdot \mathbf{Controls}_{i,w} + \varepsilon_{i,w,l,k,j}\end{aligned} \tag{3.1}$$

Where i = individual observation, w = Wave (W2 for explanatory variables, W3 for dependent variable), l = Subject (Maths, English), k = Primary caregiver (PCG), j = total number of noncognitive measures, β_C = coefficient for cognitive ability, β_{Nj} = coefficient for the j-th noncognitive measure, γ_j = coefficient for the interaction between cognition and the j-th noncognitive measure, δ' = vector of coefficients for control variables.

C. Components of the Production Function

1. Cognitive Ability

$$\text{Cognition}_{i,w} = \text{PC}(\text{Naming ability}_{i,w}, \text{Maths ability}_{i,w}, \text{Vocabulary ability}_{i,w}) \tag{3.2}$$

2. Noncognitive Measures

a) Behavioural and Emotional Characteristics (SDQ):

$$\text{NonCognition}_{i,w,k,j} \text{ for } j \in \{1,2,3,4\} = \begin{cases} \text{SDQ - Emotional Resilience}_{i,w,k} \\ \text{SDQ - Good Conduct}_{i,w,k} \\ \text{SDQ - Focused Behaviour}_{i,w,k} \\ \text{SDQ - Positive Peer Relationships}_{i,w,k} \end{cases}$$

b) Personality Traits (TIPI):

$$\text{NonCognition}_{i,w,k,j} \text{ for } j \in \{5,6,7,8,9\} = \begin{cases} \text{TIPI - Agreeable}_{i,w,k} \\ \text{TIPI - Conscientious}_{i,w,k} \\ \text{TIPI - Emotional Stability/Neuroticism}_{i,w,k} \\ \text{TIPI - Extravert}_{i,w,k} \\ \text{TIPI - Openness}_{i,w,k} \end{cases}$$

3. Control Variables

1. Socioeconomic Status: Gender, Parental education, Income quantile
2. School Characteristics: Mixed schools, DEIS schools, Fee-paying schools

4. Estimation Strategy

I estimate the linear production function using Ordinary Least Squares (OLS) regression. I chose to standardize the variables of interest to have a mean of zero and a standard deviation of one. This approach is widely recognized in econometrics for its multiple benefits (Greene, 2003; Wooldridge, 2015). By employing this technique, the benefits are two-fold: it provides a more straightforward interpretation of the results while also potentially improving the overall fit of the model (Wooldridge, 2015).

I estimate three models for each subject (Maths and English):

1. Base model (1): Includes only cognitive ability and noncognitive measures
2. Full model (2): Adds socioeconomic and school characteristic controls
3. Interaction model (3): Incorporates interaction terms between cognitive ability and noncognitive measures

This stepwise approach allows us to observe how the connections between variables change as we add more complexity to the model.

5. Interpretation of Results

The coefficients in the models can be interpreted as follows:

- β_C: The change in academic performance associated with a one standard deviation increase in cognitive ability.
- β_{Nj}: The change in academic performance associated with a one standard deviation increase in the j-th noncognitive measure.
- γ_j: The change in the effect of cognitive ability on academic performance for a one standard deviation increase in the j-th noncognitive measure. A positive value indicates complementarity, while a negative value suggests substitutability.

I use the R-squared (R^2) statistic to assess the overall explanatory power of the models and how it changes as I add more variables and interactions.

6. Limitations and Considerations

While the linear production function approach provides new perspectives about the relationships between cognitive abilities, noncognitive skills, and academic performance, it is important to acknowledge some limitations:

1. The linear form assumes constant returns to scale, which may not always hold in educational contexts.
2. The model assumes additive effects, which might oversimplify nuanced links between variables.
3. Endogeneity concerns, such as omitted variable bias or reverse causality, could affect the interpretation of the results.

In the following results section, I will present and discuss the findings from these estimations, considering both the statistical significance and practical importance of the estimated coefficients.

D. Results

Tables 3 and 4 present the regression results for the effects of cognitive ability and noncognitive skills on Junior Certificate Maths and English scores. The dependent variable (academic performance) is measured on a scale from 2 to 12 for Maths and from 5 to 12 for English, while all independent variables are standardized.

Cognitive Skills Across all models, cognitive ability emerges as the strongest predictor of academic performance. For Maths, a one standard deviation increase in cognitive ability is associated with an increase of 0.72 to 0.84 points on the 2-12 scale, depending on the model specification. For English, the effect is somewhat smaller, ranging from 0.45 to 0.50 points. This difference suggests that cognitive skills may play a more crucial role in Maths performance compared to English.

Noncognitive Skills While less impactful than cognitive skills, several noncognitive factors show significant associations with academic performance:

- **SDQ Measures (Table 4)**: Focused Behaviour is the strongest noncognitive predictor, with a one standard deviation increase associated with a 0.22 point increase in Maths scores and a 0.18 point increase in English scores (Model 3). Good Conduct and Emotional Resilience show smaller but significant positive effects on Maths scores (0.07 and 0.05 points respectively), while Positive Peer Relationships is significantly associated with English scores (0.06 points).

- **TIPI Measures (Table 3)**: Conscientiousness demonstrates the strongest effect among personality traits, with a one standard deviation increase associated with a 0.14 point increase in Maths scores and a 0.08 point increase in English scores (Model 3). Emotional Stability shows a modest but significant positive effect on Maths scores (0.06 points), while Agreeableness has a small positive effect on English scores (0.04 points).

Interaction Effects

- For SDQ measures, the interaction between Cognition and Focused Behaviour is negative and significant for both Maths (-0.06) and English (-0.07). This suggests that the positive effect of Focused Behaviour on academic performance is somewhat reduced for students with higher cognitive ability.

- For TIPI measures, the interaction between Cognition and Emotional Stability is negative and significant for Maths (-0.07), indicating that the effect of Emotional Stability on Maths performance is less pronounced for students with higher cognitive ability.

These negative interactions suggest that noncognitive skills may be particularly important for students with lower cognitive abilities, potentially offering a compensatory mechanism.

Gender Differences The Male variable shows a consistent negative and highly significant coefficient across all models. The effect is substantially larger for English (-0.44 points) compared to Maths (-0.12 points). This suggests that, when controlling for cognitive ability, noncognitive skills, and socioeconomic factors, boys tend to perform worse than girls, particularly in English. The smaller gender gap in Maths, coupled with the observation that boys score higher on average in Maths in the sample, suggests a wider distribution of Maths scores among boys.

Model Fit Examining the changes in R^2 across models provides insight into the explanatory power of different factors:

- For Maths (Table 4), R^2 increases from 0.476 in the base model to 0.503 when including noncognitive factors and controls, and further to 0.506 with interaction terms. This indicates that noncognitive factors and controls explain an additional 2.64% of the variance in Maths scores, while interaction terms contribute a further 0.32%.

- For English (Table 4), the pattern is similar but with larger increases: from 0.310 to 0.347 (3.68% increase) and then to 0.354 (0.70% increase).

While these increases in R^2 are statistically significant, their practical significance should be considered in the context of educational interventions. The larger increase for English suggests that noncognitive factors may play a more substantial role in explaining variability in English performance compared to Maths.

Subject Differences

- Cognitive skills have a stronger association with Maths performance compared to English. This may reflect the more structured and sequential nature of mathematical knowledge, which could be more closely tied to cognitive processing abilities.
- Noncognitive skills, particularly Focused Behaviour and Conscientiousness, show significant effects on both subjects, but their relative importance seems higher for English. This could suggest that success in language-related tasks may rely more heavily on self-regulation and persistent effort.
- The gender gap is more pronounced in English than in Maths, which aligns with international trends but raises questions about the factors driving this disparity in the Irish context.

These subject-specific patterns suggest that the production function for academic achievement varies across disciplines, potentially reflecting differences in how these subjects are taught, learned, and assessed. The stronger role of cognitive skills in Maths achievement compared to English may indicate that Maths skills are more dependent on formal instruction and cognitive development, while English skills might be more influenced by broader environmental and noncognitive factors.

In conclusion, while cognitive abilities remain the strongest predictors of academic performance, noncognitive factors provide meaningful additional explanatory power, especially for English performance. The detailed interactions between cognitive and noncognitive skills, as well as the observed gender and subject differences, highlight the multifaceted nature of academic achievement and suggest the need for nuanced, targeted approaches in educational policy and practice.

Table 3: Regression results: Effects of cognitive ability and personality traits (TIPI scores) on Junior Certificate Maths and English scores. TIPI scores reported by primary caregivers in Wave 2; Junior Cert scores from Wave 3. Models (1)-(3) show progressively added controls and interactions.

	Maths			English		
Model:	(1)	(2)	(3)	(1)	(2)	(3)
Variables						
Constant	9.571***	9.737***	9.751***	10.13***	10.41***	10.42***
	(0.0171)	(0.0355)	(0.0356)	(0.0151)	(0.0305)	(0.0306)
Cognition	0.8351***	0.7154***	0.7176***	0.5039***	0.4464***	0.4483***
	(0.0127)	(0.0161)	(0.0161)	(0.0113)	(0.0138)	(0.0138)
Agreeableness	0.0092	0.0290	0.0309	0.0271*	0.0349*	0.0369**
	(0.0181)	(0.0210)	(0.0210)	(0.0160)	(0.0180)	(0.0181)
Conscientiousness	0.1580***	0.1325***	0.1371***	0.0912***	0.0755***	0.0801***
	(0.0181)	(0.0210)	(0.0211)	(0.0160)	(0.0180)	(0.0181)
Emotional stability	0.0677***	0.0474**	0.0580***	0.0131	0.0045	0.0091
	(0.0184)	(0.0210)	(0.0211)	(0.0163)	(0.0180)	(0.0182)
Extraversion	-0.0191	-0.0200	-0.0181	0.0181	0.0124	0.0147
	(0.0177)	(0.0203)	(0.0204)	(0.0156)	(0.0174)	(0.0175)
Openness	-0.0725***	-0.0211	-0.0219	0.0065	0.0106	0.0138
	(0.0179)	(0.0207)	(0.0208)	(0.0159)	(0.0178)	(0.0179)
Male		-0.1148***	-0.1167***		-0.4452***	-0.4443***
		(0.0401)	(0.0400)		(0.0344)	(0.0343)
Cognition × Agree.			-0.0173			-0.0120
			(0.0161)			(0.0140)
Cognition × Consc.			-0.0136			-0.0185
			(0.0161)			(0.0138)
Cognition × Emot.			-0.0678***			-0.0262*
			(0.0156)			(0.0134)
Cognition × Extra.			0.0021			-0.0074
			(0.0149)			(0.0129)
Cognition × Open.			0.0047			-0.0234*
			(0.0154)			(0.0134)
Controls						
SES	No	Yes	Yes	No	Yes	Yes
School	No	Yes	Yes	No	Yes	Yes
Fit statistics						
Observations	5,654	3,801	3,801	5,637	3,787	3,787
R^2	0.45811	0.48680	0.49051	0.28163	0.33165	0.33481
Adjusted R^2	0.45753	0.48504	0.48809	0.28086	0.32935	0.33163

IID standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*
Agree.: Agreeableness, Consc.: Conscientiousness, Emot.: Emotional stability, Extra.: Extraversion, Open.: Openness

Note: Standardized coefficients reported. Model (1) includes only cognitive and TIPI variables; Model (2) adds SES and school controls; Model (3) includes interaction terms. TIPI: Ten-Item Personality Inventory. Cognition is a composite measure of cognitive abilities.

Table 4: Regression results: Effects of cognitive ability and behavioural characteristics (SDQ scores) on Junior Certificate Maths and English scores. SDQ scores reported by primary caregivers in Wave 2; Junior Cert scores from Wave 3. Models (1)-(3) show progressively added controls and interactions.

	Maths			English		
Model:	(1)	(2)	(3)	(1)	(2)	(3)
Variables						
Constant	9.570***	9.705***	9.729***	10.13***	10.39***	10.41***
	(0.0168)	(0.0352)	(0.0355)	(0.0148)	(0.0303)	(0.0305)
Cognition	0.7830***	0.6689***	0.6703***	0.4641***	0.4161***	0.4170***
	(0.0131)	(0.0164)	(0.0164)	(0.0115)	(0.0141)	(0.0141)
Emotional Resilience	0.0499***	0.0565**	0.0462**	-0.0274	0.0120	0.0021
	(0.0192)	(0.0226)	(0.0227)	(0.0169)	(0.0194)	(0.0194)
Good Conduct	0.0865***	0.0716***	0.0740***	0.0042	-0.0220	-0.0183
	(0.0196)	(0.0233)	(0.0234)	(0.0172)	(0.0201)	(0.0201)
Focused Behaviour	0.2415***	0.2267***	0.2166***	0.2440***	0.1902***	0.1795***
	(0.0198)	(0.0235)	(0.0236)	(0.0174)	(0.0203)	(0.0202)
Positive Peer Relationships	0.0179	-0.0031	0.0014	0.0736***	0.0503***	0.0573***
	(0.0184)	(0.0215)	(0.0218)	(0.0162)	(0.0185)	(0.0188)
Male		-0.0702*	-0.0693*		-0.4029***	-0.4007***
		(0.0402)	(0.0402)		(0.0346)	(0.0345)
Cognition × E.R.			-0.0316**			-0.0176
			(0.0161)			(0.0139)
Cognition × G.C.			0.0126			0.0266*
			(0.0181)			(0.0157)
Cognition × F.B.			-0.0565***			-0.0735***
			(0.0171)			(0.0148)
Cognition × P.P.R.			-0.0134			-0.0264**
			(0.0148)			(0.0128)
Controls						
SES	No	Yes	Yes	No	Yes	Yes
School	No	Yes	Yes	No	Yes	Yes
Fit statistics						
Observations	5,664	3,805	3,805	5,647	3,791	3,791
R^2	0.47612	0.50254	0.50572	0.31011	0.34688	0.35392
Adjusted R^2	0.47566	0.50096	0.50363	0.30950	0.34480	0.35118

IID standard-errors in parentheses
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*
E.R.: Emotional Resilience, G.C.: Good Conduct, F.B.: Focused Behaviour,
P.P.R.: Positive Peer Relationships

Note: Standardized coefficients reported. Model (1) includes only cognitive and SDQ variables; Model (2) adds SES and school controls; Model (3) includes interaction terms. SDQ: Strengths and Difficulties Questionnaire. Cognition is a composite measure of cognitive abilities. Original SDQ scales have been inverted so that higher scores indicate better outcomes.

IV. How do cognitive and noncognitive skills interact in producing academic outcomes? A nonlinear analysis

In this section I extend the traditional approach by explicitly including both cognitive and noncognitive factors as key inputs. This approach is grounded in the growing body of literature which deals with the importance of noncognitive skills in educational and life outcomes (Duckworth & Seligman, 2005; Heckman & Rubinstein, 2001). In this section I allow for a more flexible form of production function: the transcendental logarithmic production function, first introduced by Christensen, Jorgenson, and Lau in 1971. It was formally presented in their paper titled "Conjugate Duality and the Transcendental Logarithmic Production Function" which appeared in Econometrica in 1973.

I model the educational production function using a two-input a translog function. In this specification C is the variable cognition and N is a noncognitive variable. In relation to the scales used, N is either the variable Focused Behaviour (SDQ) or Conscientiousness (TIPI). I chose these two because they were the most significant factors from the previous analysis. A more restrictive form, the Cobb-Douglas with two and three inputs, can be found in the Appendix. The Cobb-Douglas model highlighted the dominant role of cognition and the significant but smaller impact of noncognitive skills, with gender differences also present. Boys showed slightly stronger cognition and girls exhibited stronger noncognitive effects, especially in Maths. The model revealed decreasing returns to scale in the production of academic achievement, with Marginal Rates of Technical Substitution indicating that more than one unit of noncognitive skill is typically needed to substitute for one unit of cognition. While these findings are informative, the Cobb-Douglas model's assumption of constant elasticity of substitution limits its ability to capture interesting relationships between inputs. The translog model, with its flexible functional form, allows for varying elasticities of substitution and interaction effects between inputs, potentially offering a more comprehensive understanding of the educational production function. Therefore, the main text will focus on the translog model, with the detailed Cobb-Douglas analysis available in the Appendix for interested readers.

1. Definition

The Translog production function is defined as a Taylor approximation of a CES production of the type $(\delta C^{\gamma} + (1-\delta)N^{\gamma})^{\frac{1}{\gamma}}$ when γ goes to 0. It is a flexible form that extends the Cobb-Douglas production function by including logarithms of inputs and their squares and cross-products. This particular specification allows for nuanced relationships between inputs and outputs, potentially revealing non-linear effects of cognitive and noncognitive skills on academic achievement, interactions between cognitive and noncognitive factors (which may enhance or mitigate each other's effects), and varying returns to scale for different combinations of inputs.

The translog function is here specifically defined as:

$$Y = AC^{\alpha}N^{\beta}\exp\left\{\frac{1}{2}\gamma_1\left[\ln(C)\right]^2 + \frac{1}{2}\gamma_2\left[\ln(N)\right]^2 + \gamma_{12}\ln(C)\ln(N)\right\} \tag{4.1}$$

Where:

$$\begin{aligned}
Y &: \text{Total output/Grade function/Academic achievement} \\
A &: \text{Total factor productivity or scaling factor} \\
C, N &: \text{Inputs} \\
\alpha, \beta &: \text{Exponents determining the output response to each input} \\
\gamma_1, \gamma_2, \gamma_{12} &: \text{Parameters capturing interactions and quadratic effects}
\end{aligned}$$

The parameters α, β, γ_1, γ_2, and γ_{12} capture distinct elements of the production function. Their estimated values offer a better understanding of the relationships between the inputs C and N and the output Y within the production process. γ_1, γ_2, and γ_{12} represent the second-order terms and interaction terms that capture non-linearities and interactions between inputs. Positive values of γ_1 or γ_2 indicate increasing returns to the respective input, while negative values suggest diminishing returns. The interaction term γ_{12} captures potential complementarities or substitutions between cognitive and noncognitive skills.

The following metrics are presented in their final estimation forms here and derived in detail in the Appendix.

2. Marginal products (MPs)

Marginal products (MPs) represent the change in total output resulting from a one-unit increase in a specific input while holding all other inputs constant. In the context of the translog production function, MPs are more elaborate than in the Cobb-Douglas model due to the inclusion of quadratic and interaction terms.

The marginal products for inputs C and N are derived by taking the partial derivative of the production function with respect to each input:

$$f_C = A\alpha C^{\alpha-1} N_0^{\beta} \exp\left\{\frac{1}{2}\gamma_1 [\ln(C)]^2 + \frac{1}{2}\gamma_2 [\ln(N_0)]^2 + \gamma_{12}\ln(C)\ln(N_0)\right\} \left[\gamma_1 \ln(C)\frac{1}{C} + \gamma_{12}\frac{\ln(N_0)}{C}\right] \tag{4.2}$$

$$f_N = A\beta C_0^{\alpha} N^{\beta-1} \exp\left\{\frac{1}{2}\gamma_1 [\ln(C_0)]^2 + \frac{1}{2}\gamma_2 [\ln(N)]^2 + \gamma_{12}\ln(C_0)\ln(N)\right\} \left[\gamma_2 \ln(N)\frac{1}{N} + \gamma_{12}\frac{\ln(C_0)}{N}\right] \tag{4.3}$$

These expressions show that the marginal products in the translog model depend not only on the levels of inputs C and N but also on their logarithms and the interaction between them.

3. Output elasticities (OEs)

Output elasticities measure the responsiveness of output to a change in inputs, expressed in percentage terms. In the translog production function, unlike in the Cobb-Douglas model, these elasticities are not constant but vary with the levels of inputs.

For the translog function, the output elasticities are derived by taking the partial derivative of the natural logarithm of the production function with respect to the logarithm of each input:

$$OE_C = \left.\frac{\partial \ln(Y)}{\partial \ln(C)}\right|_{N=N_0} = \alpha + \gamma_1 \ln(C) + \gamma_{12}\ln(N_0) \tag{4.4}$$

$$OE_N = \left.\frac{\partial \ln(Y)}{\partial \ln(N)}\right|_{C=C_0} = \beta + \gamma_2 \ln(N) + \gamma_{12} \ln(C_0) \tag{4.5}$$

These expressions show that the output elasticities in the translog model depend on the levels of both inputs (C and N), as well as the interaction term (γ_{12}). This allows for varying returns to scale and changing relative importance of inputs at different levels.

4. Marginal Rate of Technical Substitution

The Marginal Rate of Technical Substitution (MRTS) represents the rate at which one input can be substituted for another while maintaining the same level of output. In the context of educational production, the $MRTS_{CN}$ indicates how much noncognitive skill (N) is needed to compensate for a small decrease in cognitive skill (C) while keeping academic achievement constant.

Mathematically, MRTS is defined as the negative of the slope of the isoquant curve in input space. It can be derived from the ratio of the marginal products:

$$MRTS_{CN} = -\left.\frac{dN}{dC}\right|_{Y=Y_0} = \frac{f_C}{f_N} \tag{4.6}$$

where f_C and f_N are the marginal products of C and N respectively.

For the translog function, the MRTS can be expressed in terms of output elasticities:

$$MRTS_{CN} = \frac{f_C}{f_N} = \frac{OE_C}{OE_N} \cdot \frac{N}{C} \tag{4.7}$$

Substituting the expressions for OE_C and OE_N:

$$MRTS_{CN} = \frac{\alpha + \gamma_1 \ln(C) + \gamma_{12} \ln(N)}{\beta + \gamma_2 \ln(N) + \gamma_{12} \ln(C)} \cdot \frac{N}{C} \tag{4.8}$$

This MRTS shows how the rate at which cognitive skills can be substituted for noncognitive skills (or vice versa) varies with the levels of both inputs. It captures non-obvious relationships between inputs than the constant MRTS of the Cobb-Douglas function. The variable nature of both the elasticity of substitution and the MRTS in the translog function capture scenarios where the substitutability between skills varies at different skill levels, offering a better understanding of optimal skill development strategies.

5. Elasticity of Substitution (ES)

For the Translog production function, the elasticity of substitution is not constant but varies with the levels of inputs. It can be calculated using the formula:

$$\sigma = 1 - \frac{\partial \ln(MRTS)}{\partial \ln(C/N)} \tag{4.9}$$

Where MRTS is the Marginal Rate of Technical Substitution.

Using the output elasticities derived earlier:

$$OE_C = \alpha + \gamma_1 \ln(C) + \gamma_{12} \ln(N) \tag{4.10}$$

$$OE_N = \beta + \gamma_2 \ln(N) + \gamma_{12} \ln(C) \tag{4.11}$$

We can express the elasticity of substitution as:

$$\sigma = 2 - \frac{\gamma_1}{OE_C} + \frac{\gamma_{12}}{OE_N} + \frac{\gamma_{12}}{OE_C} - \frac{\gamma_2}{OE_N} \tag{4.12}$$

6. Results

A striking feature of the results is the subject-specific dynamics of skill influence. In the full sample, cognitive skills (α) show a strong and significant influence on both Maths and English performance. For Maths, α is 0.79 for SDQ and 0.83 for TIPI, while for English, it is 0.41 for SDQ and 0.45 for TIPI. This indicates a consistently strong role of cognitive skills, with a notably larger impact on Maths performance.

Noncognitive skills (β) also demonstrate significant contributions, albeit smaller than cognitive skills. In the SDQ models, β is 0.11 for Maths and 0.09 for English, while in the TIPI models, it is 0.04 for Maths and 0.02 for English. This again suggests that the SDQ measure captures aspects of noncognitive skills more relevant to academic performance than the TIPI measure.

The interaction term (γ_{12}) is negative and significant for the SDQ models in both subjects ($\gamma_{12} = -0.13$ for Maths and -0.12 for English). This negative interaction indicates a substitutive relationship between cognition and noncognition, confirming the earlier hypothesis. However, the relatively small absolute values suggest this substitutive effect is modest.

Gender differences are evident and meaningful. For girls, cognitive skills have a slightly stronger impact on Maths performance ($\alpha = 0.778$ for SDQ, 0.818 for TIPI) compared to boys ($\alpha = 0.806$ for SDQ, 0.853 for TIPI). In English, boys show a slightly higher cognitive impact ($\alpha = 0.468$ for SDQ, 0.510 for TIPI) compared to girls ($\alpha = 0.414$ for SDQ, 0.442 for TIPI). Noncognitive skills (β) show higher values for girls in both subjects, particularly for the SDQ measure ($\beta = 0.123$ for Maths, 0.076 for English) compared to boys ($\beta = 0.086$ for Maths, 0.071 for English). The interaction term (γ_{12}) shows significant negative values for both genders in most models, with generally larger absolute values for girls.

We see again that noncognitive skills play a more substantial role in girls' academic performance compared to boys'. The more pronounced substitutive relationship between cognitive and noncognitive skills for girls potentially indicates different dynamics in skill utilization between genders.

MPs for both cognitive and noncognitive skills are generally higher in Maths, indicating that incremental improvements in either skill type yield greater returns in this subject. OEs consistently show that cognitive skills have a larger impact on both Maths and English scores compared to noncognitive skills, particularly for Maths. This aligns with my previous findings from the Cobb-Douglas production estimation.

ES values vary widely across models and subsamples, suggesting that the substitutability between these skills is highly context-dependent, influenced by factors such as subject, gender, and the specific noncognitive measure used. The MRTS values, generally below 1, indicate that multiple units of noncognitive skills are typically needed to substitute for one unit of cognitive skills to maintain the same level of academic performance.

The choice of measurement tool for noncognitive skills is important. The SDQ measure consistently shows stronger relationships with academic outcomes compared to TIPI across all models, which suggests that the SDQ captures noncognitive traits more directly relevant to academic settings, potentially offering a more accurate representation of skills that influence school performance. These non-linear and non-obvious patterns provided by the estimation of the translog model offer a more subtle understanding of the educational production process than simpler models.

Table 5: Translog production function estimates for Maths achievement: comparison of TIPI and SDQ models across full sample and gender subgroups

	Maths			
Parameter	Full Sample	Boys	Girls	Δ (Boys - Girls)
TIPI Model				
A	9.43***	9.349***	9.511***	-0.162
	(0.03)	(0.042)	(0.042)	
α (Cognition)	0.83***	0.853***	0.818***	0.035
	(0.02)	(0.021)	(0.023)	
β (Conscientiousness)	0.04***	0.037***	0.048***	-0.011
	(0.01)	(0.007)	(0.008)	
γ_1	0.10	0.312*	-0.154	0.466
	(0.11)	(0.183)	(0.157)	
γ_2	0.05***	0.032*	0.069***	-0.037
	(0.01)	(0.019)	(0.020)	
γ_{12}	-0.04	-0.092**	0.004	-0.096
	(0.03)	(0.037)	(0.036)	
Marginal Product (Cognition)	0.08	0.080	0.078	0.002
Marginal Product (Conscientiousness)	0.08	0.073	0.091	-0.018
Output Elasticity (Cognition)	0.83	0.859	0.820	0.039
Output Elasticity (Conscientiousness)	0.04	0.036	0.048	-0.012
Elasticity of Substitution	-0.30	-1.941	0.853	-2.794
MRTS	0.93	1.102	0.858	0.244
SDQ Model				
A	9.45***	9.350***	9.537***	-0.187
	(0.02)	(0.035)	(0.034)	
α (Cognition)	0.79***	0.806***	0.778***	0.028
	(0.02)	(0.021)	(0.023)	
β (Focused Behaviour)	0.11***	0.086***	0.123***	-0.037
	(0.01)	(0.010)	(0.013)	
γ_1	0.20*	0.370**	0.038	0.332
	(0.11)	(0.184)	(0.158)	
γ_2	0.06***	0.046***	0.083***	-0.037
	(0.01)	(0.012)	(0.016)	
γ_{12}	-0.13***	-0.112***	-0.149***	0.037
	(0.03)	(0.041)	(0.052)	
Marginal Product (Cognition)	0.07	0.076	0.074	0.002
Marginal Product (Focused Behaviour)	0.12	0.103	0.140	-0.037
Output Elasticity (Cognition)	0.79	0.813	0.778	0.035
Output Elasticity (Focused Behaviour)	0.11	0.084	0.125	-0.041
Elasticity of Substitution	-0.26	-0.470	-0.092	-0.378
MRTS	0.61	0.733	0.530	0.203
Observations	5,631	2,667	2,801	

*Standard errors in parentheses. Signif. Codes: ***: 0.001, **: 0.01, *: 0.05*

Note: The table displays estimates for the Translog production function applied to Maths scores. The TIPI Model focuses on Cognition and Conscientiousness, while the SDQ Model considers Cognition and Focused Behaviour. MRTS: Marginal Rate of Technical Substitution.

Table 6: Translog production function estimates for English achievement: comparison of TIPI and SDQ models across full sample and gender subgroups

	English			
Parameter	Full Sample	Boys	Girls	Δ (Boys - Girls)
TIPI Model				
A	10.09***	9.837***	10.363***	-0.526
	(0.03)	(0.037)	(0.034)	
α (Cognition)	0.45***	0.510***	0.442***	0.068
	(0.01)	(0.017)	(0.017)	
β (Conscientiousness)	0.02***	0.020***	0.017***	0.003
	(0.00)	(0.006)	(0.006)	
γ_1	-0.25***	-0.248	-0.318***	0.070
	(0.09)	(0.153)	(0.116)	
γ_2	0.03***	0.027*	0.016	0.011
	(0.01)	(0.016)	(0.015)	
γ_{12}	-0.03	-0.038	-0.008	-0.030
	(0.02)	(0.031)	(0.027)	
Marginal Product (Cognition)	0.05	0.049	0.047	0.002
Marginal Product (Conscientiousness)	0.05	0.041	0.035	0.006
Output Elasticity (Cognition)	0.45	0.506	0.447	0.059
Output Elasticity (Conscientiousness)	0.02	0.019	0.017	0.002
Elasticity of Substitution	0.16	-0.953	1.257	-2.210
MRTS	0.89	1.217	1.342	-0.125
SDQ Model				
A	10.08***	9.830***	10.337***	-0.507
	(0.02)	(0.030)	(0.027)	
α (Cognition)	0.41***	0.468***	0.414***	0.054
	(0.01)	(0.018)	(0.017)	
β (Focused Behaviour)	0.09***	0.071***	0.076***	-0.005
	(0.01)	(0.008)	(0.010)	
γ_1	-0.17*	-0.141	-0.166	0.025
	(0.09)	(0.154)	(0.117)	
γ_2	0.06***	0.047***	0.063***	-0.016
	(0.01)	(0.010)	(0.012)	
γ_{12}	-0.12***	-0.118***	-0.152***	0.034
	(0.02)	(0.034)	(0.038)	
Marginal Product (Cognition)	0.04	0.045	0.043	0.002
Marginal Product (Focused Behaviour)	0.11	0.088	0.095	-0.007
Output Elasticity (Cognition)	0.41	0.466	0.416	0.050
Output Elasticity (Focused Behaviour)	0.09	0.069	0.078	-0.009
Elasticity of Substitution	0.02	-0.348	-0.722	0.374
MRTS	0.38	0.513	0.455	0.058
Observations	5,631	2,667	2,801	

Standard errors in parentheses.
*Signif. Codes: ***: 0.001, **: 0.01, *: 0.05*

Note: The table displays estimates for the Translog production function applied to English scores. The TIPI Model focuses on Cognition and Conscientiousness, while the SDQ Model considers Cognition and Focused Behaviour. MRTS: Marginal Rate of Technical Substitution.

V. Conclusion

This study contributes to our understanding of educational outcomes by examining the joint effects and interactions of cognitive and noncognitive skills, with a focus on gender differences and academic achievement in Maths and English. Using data from the Growing Up in Ireland longitudinal study, I employed linear and translog production functions to quantify the relative impacts of these factors on academic performance.

Across all models, cognitive ability emerged as the primary factor influencing academic performance in both Maths and English. Its impact was more pronounced in Maths, with scores increasing by 0.67 to 0.84 points per standard deviation of cognitive ability, compared to 0.42 to 0.50 points for English. This highlights the central role of cognitive development, especially in Maths.

Noncognitive factors also played an important role, though their impact was generally smaller. Among these, Focused Behaviour (measured by the SDQ) was the most significant noncognitive predictor, positively affecting Maths scores by 0.22 points and English scores by 0.18 points. Conscientiousness (assessed using the TIPI) had the strongest effect among personality traits, contributing 0.14 points to Maths scores and 0.08 points to English scores. Other factors, such as Good Conduct, Emotional Resilience (SDQ), and Emotional Stability (TIPI), had smaller but still significant positive effects, particularly on Maths. Overall, both cognitive and noncognitive factors had a stronger influence on Maths compared to English. This might be due to the greater reliance on formal instruction and cognitive development in Maths, as noted by Duckworth and Yeager (2015).

In terms of gender differences, boys demonstrated slightly higher cognitive output elasticities in both subjects, while girls showed stronger effects of noncognitive factors, particularly in Maths. These differences may stem from a combination of socialization patterns, potential biological differences in cognitive development, and varying interactions with teaching methods and educational environments (Hyde, 2016). The higher returns for both skill types in Maths could be attributed to the subject's cumulative nature, where each improvement builds more directly on previous knowledge(Rittle-Johnson et al., 2015).

My analysis using the translog model also accounted for non-linear effects and yielded a variable elasticity of substitution. Specifically, the degree of substitutability between these skills varies depending on the student's proficiency, a finding that was not apparent in the simpler Cobb-Douglas specification. This variation in substitutability suggests that the relative importance of cognitive versus noncognitive skills may shift as students progress academically.

Both models' estimation provided evidence of decreasing returns to scale in educational production, meaning that proportional increases in all inputs result in less than proportional increases in academic performance. This finding, coupled with the observed gender and subject differences, indicates that a ”one-size-fits-all” approach to improving educational outcomes is likely sub-optimal. Instead, targeted interventions that consider the unique contexts of different student populations may be more effective.

My findings can also be explained by a few factors. The stronger impact of cognitive skills could reflect their direct involvement in understanding and applying academic content. The subject-specific differences might arise from the distinct nature of Maths (requiring sustained, focused attention) versus language learning (involving a broader range of skills). The compensation patterns could be due to the traditional emphasis on cognitive abilities in educational settings. Gender differences might also relate to societal expectations, teaching methods, or developmental patterns. The greater variability in English performance could stem from the subject's multifaceted nature, allowing for more diverse combinations of skills to influence outcomes.

The results from this study have important implications for educational policy and practice. While cognitive skills remain central, the non-trivial role of noncognitive factors suggests that interventions targeting these skills could yield meaningful improvements in academic performance, especially when tailored to specific subjects and genders. This aligns with the findings of Durlak et al. (2011) on the effectiveness of social and emotional learning programs. Based on my results, I think that boys might benefit from additional support in language skills and noncognitive areas like Focused Behaviour, while girls might benefit from interventions aimed at boosting confidence and performance in Maths, as also suggested by Dweck (2007)'s work on mindset interventions.

In conclusion, this study demonstrates that academic achievement is a dynamic process influenced by a myriad of factors, with cognitive skills playing a dominant but not exclusive role. As we move forward, it will be important to continue refining our understanding of these complex relationships and translating these findings into effective educational practices. By doing so, we can work towards an educational system that not only promotes academic achievement but also equips students with the diverse skills and abilities they need to thrive in an increasingly demanding and dynamic world.

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VI. Appendices

In these Appendices, I provide supplementary information, detailed mathematical derivations, and additional analyses that support and extend the main study presented in this paper. My aim is to offer readers a deeper understanding of the methodological and theoretical foundations underpinning my research on the connection between cognitive and noncognitive skills and educational outcomes.

1. I begin by presenting the full Ten-Item Personality Inventory (TIPI) and the Strengths and Difficulties Questionnaire (SDQ), which I used to assess noncognitive skills. I explain each instrument in detail, including their scoring methods and relevance to my study.

2. I then present a ”Concise version” of the more detailed Appendix that follows.

3. Next, I elaborate on various production function specifications used in my analysis. I provide derivations for both two-input and three-input versions of the Cobb-Douglas production function, including marginal products, output elasticities, elasticity of substitution, and marginal rates of technical substitution.

4. I then present the translog production function, discussing its advantages as a more flexible functional form and providing detailed derivations of its key metrics.

5. I also explore the Constant Elasticity of Substitution (CES) production function, examining both two-input and three-input versions of this more general form, which encompasses the Cobb-Douglas as a special case.

6. For each production function, I provide comprehensive derivations of marginal products, output elasticities, elasticities of substitution, marginal rates of technical substitution, returns to scale, and, for CES functions, isoquants.

7. I also include additional analyses, such as a comparison between two-input and three-input CES models, a discussion of nested CES functions, economic interpretation of parameters, and policy implications of different model specifications.

Finally, I discuss the limitations of my approach and important empirical considerations for interpreting and applying these models.

A. TIPI questionnaire

The TIPI is a concise personality assessment tool designed to measure the Big Five personality traits. Developed by Gosling, Rentfrow, and Swann in 2003, it serves as a rapid alternative to more extensive personality inventories. The TIPI consists of just ten items, with two items dedicated to each of the five major personality dimensions: Extraversion, Agreeableness, Conscientiousness, Emotional Stability (the inverse of Neuroticism), and Openness to Experience. This inventory is particularly useful in research settings where time constraints are significant or when personality assessment is not the primary focus of the study. It asks respondents to rate themselves, or the study’s child in the Growing up in Ireland, on a series of paired traits using a 7-point scale, ranging from ”Disagree strongly” to ”Agree strongly”. For example, Extraversion is assessed through items like ”Extraverted, enthusiastic” and its reverse-coded counterpart ”Reserved, quiet”. While the TIPI ”sacrifices” some degree of reliability and validity compared to longer measures, it still provides a reasonable approximation of an individual's personality profile. Its brevity makes it an attractive option for large-scale surveys, online studies, or situations where a quick personality snapshot is needed. However, researchers and practitioners are aware of its limitations and use it judiciously, understanding that it offers a broad-brush picture rather than a nuanced personality portrait.

Disagree strongly	Disagree moderately	Disagree a little	Neither disagree nor agree	Agree a little	Agree moderately	Agree strongly
1	2	3	4	5	6	7

Characteristic	Grade (1-7)
1. Extraverted, enthusiastic	______
2. Critical, quarrelsome	______
3. Dependable, self-disciplined	______
4. Anxious, easily upset	______
5. Open to new experiences, complex	______
6. Reserved, quiet	______
7. Sympathetic, warm	______
8. Disorganized, careless	______
9. Calm, emotionally stable	______
10. Conventional, uncreative	______

Conscientiousness
Score for #3: ______
+ (8 - Score for #8): ______
= ______
Divide your answer by 2.
Conscientiousness Score = ______

Agreeableness
Score for #7: ______
+ (8 - Score for #2): ______
= ______
Divide your answer by 2.
Agreeableness Score = ______

Emotional Stability
Score for #9: ______
+ (8 - Score for #4): ______
= ______
Divide your answer by 2.
Emotional Stability Score = ______

Openness to Experience
Score for #5: ______
+ (8 - Score for #10): ______
= ______
Divide your answer by 2.
Openness to Experience Score = ______

Extraversion
Score for #1: ______
+ (8 - Score for #6): ______
= ______
Divide your answer by 2.
Extraversion Score = ______

B. SDQ questionnaire

The Strengths and Difficulties Questionnaire (SDQ), developed by Robert Goodman (1997), is a widely-used behavioural screening tool designed for children and adolescents aged 3 to 16 years. Unlike many assessment instruments that focus solely on problems, the SDQ takes a more "balanced" approach by examining both difficulties and strengths in young people's behaviour and emotional well-being. The questionnaire consists of 25 items divided into five scales: Emotional Symptoms, Conduct Problems, Hyperactivity/Inattention, Peer-relationship Problems, and Prosocial Behaviour. Such structure allows for a comprehensive evaluation of a child's psychological adjustment, covering internalizing problems, externalizing issues, and positive social behaviours. The SDQ is very versatile: it offers versions for parents, teachers, and self-report (for older children and adolescents), allowing for a multi-informant approach to assessment. This multi-perspective view can provide a more rounded understanding of a child's behaviour across different contexts. The questionnaire uses a 3-point Likert scale ("Not True", "Somewhat True", "Certainly True") for responses, making it accessible and easy to complete. It typically takes between 5 to 10 minutes to fill out, providing a balance between comprehensiveness and practicality. Internationally recognized and translated into numerous languages, the SDQ has become a valuable tool in both clinical and research settings. It is particularly useful for early identification of potential mental health issues, allowing for timely intervention. The inclusion of the Prosocial scale also provides insight into a child's positive social behaviours, offering a more holistic view of their functioning. I decided not to use this Prosocial scale because of the way the total score was calculated in the AMF files. While the SDQ is not a diagnostic tool, its scores can indicate whether a child might benefit from further assessment or support. Its widespread use also facilitates comparisons across different populations and cultures, contributing to a cross-subject understanding of child and adolescent mental health on a global scale.

Example taken from the Youth in Mind (2023) website https://sdqinfo.org/:

Strengths and Difficulties Questionnaire

For each item, please mark the box for Not True, Somewhat True or Certainly True. Answer all items as best you can even if you are not absolutely certain. Please give your answers on the basis of the child's behaviour over the last six months.

	Not True	Somewhat True	Certainly True
1. Considerate of other people's feelings	☐	☐	☐
2. Restless, overactive, cannot stay still for long	☐	☐	☐
3. Often complains of headaches, stomach-aches or sickness	☐	☐	☐
4. Shares readily with other children (treats, toys, pencils etc.)	☐	☐	☐
5. Often has temper tantrums or hot tempers	☐	☐	☐
6. Rather solitary, tends to play alone	☐	☐	☐
7. Generally obedient, usually does what adults request	☐	☐	☐
8. Many worries, often seems worried	☐	☐	☐
9. Helpful if someone is hurt, upset or feeling ill	☐	☐	☐
10. Constantly fidgeting or squirming	☐	☐	☐
11. Has at least one good friend	☐	☐	☐
12. Often fights with other children or bullies them	☐	☐	☐
13. Often unhappy, down-hearted or tearful	☐	☐	☐
14. Generally liked by other children	☐	☐	☐
15. Easily distracted, concentration wanders	☐	☐	☐
16. Nervous or clingy in new situations, easily loses confidence	☐	☐	☐
17. Kind to younger children	☐	☐	☐
18. Often lies or cheats	☐	☐	☐
19. Picked on or bullied by other children	☐	☐	☐
20. Often volunteers to help others (parents, teachers, other children)	☐	☐	☐
21. Thinks things out before acting	☐	☐	☐
22. Steals from home, school or elsewhere	☐	☐	☐
23. Gets on better with adults than with other children	☐	☐	☐
24. Many fears, easily scared	☐	☐	☐
25. Sees tasks through to the end, good attention span	☐	☐	☐

The SDQ is divided into five sections, each containing five questions:

1. **Emotional Symptoms Scale:**
   - 3. Often complains of headaches, stomach-aches or sickness
   - 8. Many worries, often seems worried
   - 13. Often unhappy, down-hearted or tearful
   - 16. Nervous or clingy in new situations, easily loses confidence
   - 24. Many fears, easily scared
2. **Conduct Problems Scale:**
   - 5. Often has temper tantrums or hot tempers
   - 7. Generally obedient, usually does what adults request (reverse scored)
   - 12. Often fights with other children or bullies them
   - 18. Often lies or cheats
   - 22. Steals from home, school or elsewhere
3. **Hyperactivity Scale:**

- 2. Restless, overactive, cannot stay still for long
- 10. Constantly fidgeting or squirming
- 15. Easily distracted, concentration wanders
- 21. Thinks things out before acting (reverse scored)
- 25. Sees tasks through to the end, good attention span (reverse scored)

4. **Peer Problems Scale:**
   - 6. Rather solitary, tends to play alone
   - 11. Has at least one good friend (reverse scored)
   - 14. Generally liked by other children (reverse scored)
   - 19. Picked on or bullied by other children
   - 23. Gets on better with adults than with other children

5. **Prosocial Scale:**
   - 1. Considerate of other people's feelings
   - 4. Shares readily with other children (treats, toys, pencils etc.)
   - 9. Helpful if someone is hurt, upset or feeling ill
   - 17. Kind to younger children
   - 20. Often volunteers to help others (parents, teachers, other children)

Note: Items marked as "reverse scored" are phrased positively, so their scores are reversed when calculating the total for that scale.

VII. Concise Appendix

A. Cobb-Douglas Production Function

1. Two-Input Model

The Cobb-Douglas production function with two inputs is:

$$Y = AC^{\alpha}N^{\beta} \tag{7.1}$$

where Y is output (academic achievement), C is cognition, N is noncognitive skill.
Marginal Products:

$$f_C = \alpha\frac{Y}{C} \tag{7.2}$$

$$f_N = \beta\frac{Y}{N} \tag{7.3}$$

Output Elasticities:

$$OE_C = \alpha \tag{7.4}$$

$$OE_N = \beta \tag{7.5}$$

2. Three-Input Model

The three-input variant is:

$$Y = AC^{\alpha} N_E^{\beta_1} N_I^{\beta_2} \tag{7.6}$$

where N_E and N_I are external and internal noncognitive skills.
Marginal Products:

$$f_C = \alpha \frac{Y}{C} \tag{7.7}$$

$$f_{N_E} = \beta_1 \frac{Y}{N_E} \tag{7.8}$$

$$f_{N_I} = \beta_2 \frac{Y}{N_I} \tag{7.9}$$

Output Elasticities:

$$OE_C = \alpha \tag{7.10}$$

$$OE_{N_E} = \beta_1 \tag{7.11}$$

$$OE_{N_I} = \beta_2 \tag{7.12}$$

B. Translog Production Function (Two-Input)

$$Y = AC^{\alpha} N^{\beta} \exp\left\{ \frac{1}{2}\gamma_1 \left[\ln(C)\right]^2 + \frac{1}{2}\gamma_2 \left[\ln(N)\right]^2 + \gamma_{12} \ln(C) \ln(N) \right\} \tag{7.13}$$

Output Elasticities:

$$OE_C = \alpha + \gamma_1 \ln(C) + \gamma_{12} \ln(N) \tag{7.14}$$

$$OE_N = \beta + \gamma_2 \ln(N) + \gamma_{12} \ln(C) \tag{7.15}$$

C. CES Production Function

1. Two-Input Model

$$Y = A\left[\alpha C^{\rho} + (1-\alpha) N^{\rho}\right]^{\frac{1}{\rho}} \tag{7.16}$$

where $\rho = \frac{\sigma - 1}{\sigma}$ and σ is elasticity of substitution.
Marginal Products:

$$f_C = A\alpha C^{\rho-1}\left[\alpha C^{\rho} + (1-\alpha) N^{\rho}\right]^{\frac{1}{\rho}-1} \tag{7.17}$$

$$f_N = A(1-\alpha) N^{\rho-1}\left[\alpha C^{\rho} + (1-\alpha) N^{\rho}\right]^{\frac{1}{\rho}-1} \tag{7.18}$$

Output Elasticities:

$$OE_C = \frac{\alpha C^{\rho}}{\alpha C^{\rho} + (1-\alpha) N^{\rho}} \tag{7.19}$$

$$OE_N = \frac{(1-\alpha) N^{\rho}}{\alpha C^{\rho} + (1-\alpha) N^{\rho}} \tag{7.20}$$

2. Three-Input Model

$$Y = A\left[\alpha C^{\rho} + (1-\alpha)\left(\beta N_E^{\rho} + (1-\beta)N_I^{\rho}\right)\right]^{\frac{1}{\rho}} \tag{7.21}$$

Output Elasticities:

$$OE_C = \frac{\alpha C^{\rho}}{\alpha C^{\rho} + (1-\alpha)\left(\beta N_E^{\rho} + (1-\beta)N_I^{\rho}\right)} \tag{7.22}$$

$$OE_{N_E} = \frac{(1-\alpha)\beta N_E^{\rho}}{\alpha C^{\rho} + (1-\alpha)\left(\beta N_E^{\rho} + (1-\beta)N_I^{\rho}\right)} \tag{7.23}$$

$$OE_{N_I} = \frac{(1-\alpha)(1-\beta) N_I^{\rho}}{\alpha C^{\rho} + (1-\alpha)\left(\beta N_E^{\rho} + (1-\beta)N_I^{\rho}\right)} \tag{7.24}$$

D. Key Properties Across All Models

1. Returns to Scale: Sum of output elasticities
2. MRTS: Ratio of marginal products
3. Elasticity of Substitution (σ):
    - $\sigma > 1$: Inputs are substitutes
    - $\sigma < 1$: Inputs are complements
    - $\sigma = 1$: Cobb-Douglas case

E. Empirical Results

1. Cobb-Douglas Estimates

Two-Input Model (Maths)

- TIPI: $\alpha_{cognition} = 0.781^{***}$, $\beta_{conscientiousness} = 0.024^{***}$
- SDQ: $\alpha_{cognition} = 0.742^{***}$, $\beta_{focused} = 0.071^{***}$
- Gender differences: Boys show higher cognitive elasticities

Two-Input Model (English)

- TIPI: $\alpha_{cognition} = 0.441^{***}$, $\beta_{conscientiousness} = 0.014^{***}$
- SDQ: $\alpha_{cognition} = 0.405^{***}$, $\beta_{focused} = 0.060^{***}$
- Lower returns to scale in English compared to Maths

2. MRTS Results

Maths

- TIPI: 1.420 units of Conscientiousness needed to substitute for 1 unit of Cognition
- SDQ: 0.786 units of Focused Behaviour needed to substitute for 1 unit of Cognition
- Gender differences more pronounced in SDQ measures

English

- TIPI: 1.390 units of Conscientiousness needed for 1 unit of Cognition
- SDQ: 0.515 units of Focused Behaviour needed for 1 unit of Cognition
- Greater variation in MRTS values compared to Maths

3. Key Findings

1. Cognition remains primary driver of academic performance
2. SDQ measures show stronger relationships than TIPI
3. Girls demonstrate stronger noncognitive effects, especially in Maths
4. Both cognitive and noncognitive factors have stronger influence on Maths vs English
5. All models show decreasing returns to scale

VIII. Extended Appendix

A. Derivation of Marginal Products, Output Elasticities and Elasticities of Substitution for a Cobb-Douglas production function with two and three inputs

For the Cobb-Douglas with two inputs:

$$f_{CC} = \frac{\partial[A\alpha C^{\alpha-1}N^{\beta}]}{\partial C} = A\alpha(\alpha-1)C^{\alpha-2}N^{\beta} = \alpha(\alpha-1)AN^{\beta}\frac{C^{\alpha}}{C^2} = \alpha(\alpha-1)\frac{Y}{C^2} \tag{8.1}$$

$$f_{NN} = \frac{\partial[A\beta C^{\alpha}N^{\beta-1}]}{\partial C} = A\beta(\beta-1)N^{\beta-2}C^{\alpha} = \beta(\beta-1)AC^{\alpha}\frac{N^{\beta}}{N^2} = \beta(\beta-1)\frac{Y}{N^2} \tag{8.2}$$

For the output elasticities of the three-input Cobb-Douglas, we start with the log-transformed function:

$$\ln(Y) = \ln(A) + \alpha\ln(C) + \beta_1\ln(N_E) + \beta_2\ln(N_I) \tag{8.3}$$

The output elasticity with respect to C is defined as:

$$OE_C = \frac{\partial Y}{\partial C}\cdot\frac{C}{Y} \tag{8.4}$$

Now we can apply the chain rule. Rewriting this as:

$$OE_C = \frac{\partial\ln(Y)}{\partial\ln(C)} = \frac{\partial Y}{\partial C}\cdot\frac{\partial\ln(C)}{\partial C}\cdot\frac{C}{Y} \tag{8.5}$$

We know that $\frac{\partial\ln(C)}{\partial C} = \frac{1}{C}$, so:

$$OE_C = \frac{\partial Y}{\partial C} \cdot \frac{1}{C} \cdot \frac{C}{Y} = \frac{\partial Y}{\partial C} \cdot \frac{1}{Y} \tag{8.6}$$

Which is equivalent to:

$$OE_C = \frac{\partial \ln(Y)}{\partial \ln(C)} \tag{8.7}$$

From the log-transformed equation, we can directly see that this partial derivative is equal to α:

$$OE_C = \frac{\partial \ln(Y)}{\partial \ln(C)} = \alpha \tag{8.8}$$

$$\sigma_{C,N_E} = \frac{d(C/N_E)}{d(f_{N_E}/f_C)} \cdot \frac{f_{N_E}/f_C}{C/N_E} \tag{8.9}$$

$$\frac{f_{N_E}}{f_C} = \frac{\beta_1 A \cdot C^\alpha \cdot N_E^{\beta_1 - 1} \cdot N_I^{\beta_2}}{\alpha A \cdot C^{\alpha-1} \cdot N_E^{\beta_1} \cdot N_I^{\beta_2}} = \frac{\beta_1}{\alpha} \cdot \frac{C}{N_E} \tag{8.10}$$

$$\frac{d(f_{N_E}/f_C)}{d(C/N_E)} = \frac{\beta_1}{\alpha} \tag{8.11}$$

Substituting into the elasticity formula:

$$\sigma_{C,N_E} = \frac{1}{\beta_1/\alpha} \cdot \frac{(\beta_1/\alpha) \cdot (C/N_E)}{C/N_E} = 1 \tag{8.12}$$

Elasticity of substitution between C and N_I:

The calculation is similar to the above, replacing N_E with N_I and β_1 with β_2:

$$\sigma_{C,N_I} = \frac{d(C/N_I)}{d(f_{N_I}/f_C)} \cdot \frac{f_{N_I}/f_C}{C/N_I} = 1 \tag{8.13}$$

Elasticity of substitution between N_E and N_I:

$$\sigma_{N_E,N_I} = \frac{d(N_E/N_I)}{d(f_{N_I}/f_{N_E})} \cdot \frac{f_{N_I}/f_{N_E}}{N_E/N_I} \tag{8.14}$$

$$\frac{f_{N_I}}{f_{N_E}} = \frac{\beta_2 A \cdot C^\alpha \cdot N_E^{\beta_1} \cdot N_I^{\beta_2 - 1}}{\beta_1 A \cdot C^\alpha \cdot N_E^{\beta_1 - 1} \cdot N_I^{\beta_2}} = \frac{\beta_2}{\beta_1} \cdot \frac{N_E}{N_I} \tag{8.15}$$

$$\frac{d(f_{N_I}/f_{N_E})}{d(N_E/N_I)} = \frac{\beta_2}{\beta_1} \tag{8.16}$$

Substituting into the elasticity formula:

$$\sigma_{N_E,N_I} = \frac{1}{\beta_2/\beta_1} \cdot \frac{(\beta_2/\beta_1) \cdot (N_E/N_I)}{N_E/N_I} = 1 \tag{8.17}$$

B. Cobb-Douglas with three inputs

We begin with a Cobb-Douglas production function incorporating three inputs:

$$Y = f(C, N_E, N_I) = AC^{\alpha} N_E^{\beta_1} N_I^{\beta_2} \tag{8.18}$$

Where:

$$\begin{aligned} Y &: \text{Total output/Grade function/Academic achievement} \\ A &: \text{Total factor productivity/scaling factor} \\ C &: \text{Input representing cognition} \\ N_E, N_I &: \text{Inputs representing noncognitive measures} \\ \alpha, \beta_1, \beta_2 &: \text{Exponents determining the output response to each input} \end{aligned}$$

This function assumes a Cobb-Douglas form, where the exponents α, β_1, and β_2 indicate the elasticity or the degree of influence of each input on the total output Y. C is a measure of cognition and N_E is a noncognitive measure that I call External Control and N_I is also a noncognitive measure which I call Internal Control. In relation to the scales used (TIPI and SDQ), Internal Control is a proxy for Focused Behaviour and Conscientiousness, and External Control is a proxy for Emotional Resilience and Emotional Stability. These are the four variables that seem to appear to be the most significant ones. I also calculated the correlation matrix for all nine variables (four for the SDQ and five for the TIPI scale), and these pairs had the highest coefficients (0.407 and 0.409, respectively). The use of separate noncognitive inputs allows us to better capture the multidimensional nature of noncognitive skills and their potentially different impacts on academic achievement. I could have used any other TIPI or SDQ sub-scale, but from the regression results, I judged the aforementioned four as the most likely to influence test scores/academic achievement.

1. Marginal products (MPs)

Marginal products (MPs) represent the change in total output resulting from a one-unit increase in a specific input while holding all other inputs constant.

Taking the derivative of the marginal product of capital wrt capital (i.e., taking the second derivative of the production function wrt capital), we have:

$$f_C = \left.\frac{\partial f}{\partial C}\right|_{N_E = NE0, N_I = N_{I0}} = A\alpha C^{\alpha-1}(N_{E0})^{\beta_1}(N_{I0})^{\beta_2} \tag{8.19}$$

$$f_{N_E} = \left.\frac{\partial f}{\partial N_E}\right|_{C = C_0, N_I = NI0} = AC_0^{\alpha}\beta_1(N_E)^{\beta_1-1}(N_{I0})^{\beta_2} \tag{8.20}$$

$$f_{N_I} = \left.\frac{\partial f}{\partial N_I}\right|_{C = C_0, N_E = NE0} = AC_0^{\alpha}(N_{E0})^{\beta_1}\beta_2(N_I)^{\beta_2-1} \tag{8.21}$$

2. Output elasticities (OEs)

Output elasticities measure the responsiveness of output to a change in an input, expressed in percentage terms.

Given:

$$\ln(Y) = \ln(A) + \alpha \ln(C) + \beta_1 \ln(N_E) + \beta_2 \ln(N_I) \tag{8.22}$$

With output elasticities defined as:

$$OE_C = \left.\frac{\partial \ln(Y)}{\partial \ln(C)}\right|_{N_E=N_{E0},N_I=N_{I0}} = \alpha \tag{8.23}$$

$$OE_{N_E} = \left.\frac{\partial \ln(Y)}{\partial \ln(N_E)}\right|_{C=C_0,N_I=N_{I0}} = \beta_1 \tag{8.24}$$

$$OE_{N_I} = \left.\frac{\partial \ln(Y)}{\partial \ln(N_I)}\right|_{C=C_0,N_E=N_{E0}} = \beta_2 \tag{8.25}$$

The scale elasticity (SCE) measures by the percent change in output from a simultaneous 1% change in all inputs, then:

$$SCE = OE_C + OE_{N_E} + OE_{N_I} = \alpha + \beta_1 + \beta_2 \tag{8.26}$$

3. Elasticity of substitution (ES)

The elasticity of substitution (ES, σ) is defined as the degree to which the marginal rate of substitution between two inputs varies as the ratio of the quantity of those inputs varies while output is held constant (Stern, 2009):

$$\sigma = \frac{\frac{d(X/Y)}{(X/Y)}}{\frac{d(MP_X/MP_Y)}{(MP_X/MP_Y)}} \tag{8.27}$$

Where $MP_X(f_X)$ and $MP_Y(f_Y)$ are the marginal products of X and Y respectively. For the Cobb-Douglas production function, the σ between any two inputs is always = 1. This is a key property of the Cobb-Douglas function. To illustrate this for the three-input case:

$$\frac{f_C}{f_{N_E}} = \frac{\alpha f/C}{\beta_1 f/N_E} = \frac{\alpha N_E}{\beta_1 C} \tag{8.28}$$

If we were to calculate σ_{C,N_E}, we would find:

$$\sigma = \frac{\frac{d(C/N_E)}{(C/N_E)}}{\frac{(\alpha N_E)/(\beta_1 C)}{(\alpha N_E)/(\beta_1 C)}} = 1 \tag{8.29}$$

4. Estimation and discussion

TIPI:

$$JC_{M,E} = A(Cognition)^{\alpha}(EmotionalStability)^{\beta_1}(Conscientiousness)^{\beta_2} \tag{8.30}$$

SDQ:

$$JC_{M,E} = A(Cognition)^{\alpha}(EmotionalResilience)^{\beta_1}(FocusedBehaviour)^{\beta_2} \tag{8.31}$$

With $JC_{M,E}$ representing the score in the Junior Cert for Maths (M) and English (E).

Table 7: Cobb-Douglas Production Function Estimates for Maths Scores

Parameter	**Full Sample**	**Boys**	**Girls**	**Δ (Boys - Girls)**
TIPI Model				
A	0.257***	0.225***	0.264***	-0.039
	(0.015)	(0.019)	(0.021)	
α (Cognition)	0.777***	0.804***	0.774***	0.030
	(0.012)	(0.018)	(0.018)	
β_1 (Emotional Stability)	0.010**	0.013**	0.005	0.007
	(0.003)	(0.005)	(0.004)	
β_2 (Conscientiousness)	0.022***	0.020***	0.023***	-0.003
	(0.003)	(0.004)	(0.004)	
Marginal Product (Cognition)	0.075	0.077	0.076	0.001
Marginal Product (Emotional Stability)	0.021	0.028	0.012	0.016
Marginal Product (Conscientiousness)	0.050	0.046	0.049	-0.004
Returns to Scale	0.809	0.837	0.802	0.034
SDQ Model				
A	0.269***	0.238***	0.274***	-0.036
	(0.018)	(0.025)	(0.024)	
α (Cognition)	0.737***	0.765***	0.729***	0.036
	(0.015)	(0.023)	(0.020)	
β_1 (Emotional Resilience)	0.024**	0.031*	0.022*	0.009
	(0.008)	(0.013)	(0.010)	
β_2 (Focused Behaviour)	0.068***	0.053***	0.081***	-0.027
	(0.006)	(0.008)	(0.010)	
Marginal Product (Cognition)	0.071	0.073	0.071	0.002
Marginal Product (Emotional Resilience)	0.027	0.035	0.027	0.009
Marginal Product (Focused Behaviour)	0.086	0.072	0.098	-0.027
Returns to Scale	0.828	0.850	0.832	0.018
Observations	5,631	2,667	2,801	

Note: Standard errors in parentheses.
Significance levels: $^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table 8: Cobb-Douglas Production Function Estimates for English Scores

Parameter	**Full Sample**	**Boys**	**Girls**	**Δ (Boys - Girls)**
TIPI Model				
A	1.320***	1.020***	1.390***	-0.370
	(0.060)	(0.072)	(0.084)	
α (Cognition)	0.439***	0.490***	0.434***	0.056
	(0.010)	(0.015)	(0.013)	
β_1 (Emotional Stability)	0.004	0.007	0.003	0.004
	(0.003)	(0.004)	(0.003)	
β_2 (Conscientiousness)	0.013***	0.009**	0.010**	-0.000
	(0.002)	(0.003)	(0.003)	
Marginal Product (Cognition)	0.045	0.048	0.046	0.003
Marginal Product (Emotional Stability)	0.008	0.015	0.006	0.009
Marginal Product (Conscientiousness)	0.031	0.022	0.022	0.000
Returns to Scale	0.456	0.506	0.446	0.060
SDQ Model				
A	1.390***	1.090***	1.430***	-0.340
	(0.082)	(0.107)	(0.105)	
α (Cognition)	0.405***	0.453***	0.403***	0.050
	(0.013)	(0.022)	(0.017)	
β_1 (Emotional Resilience)	0.002	0.017	0.011	0.006
	(0.007)	(0.012)	(0.008)	
β_2 (Focused Behaviour)	0.059***	0.044***	0.051***	-0.007
	(0.005)	(0.008)	(0.008)	
Marginal Product (Cognition)	0.041	0.045	0.043	0.002
Marginal Product (Emotional Resilience)	0.003	0.020	0.013	0.006
Marginal Product (Focused Behaviour)	0.080	0.061	0.067	-0.006
Returns to Scale	0.467	0.514	0.465	0.049
Observations	5,631	2,667	2,801	

Note: Standard errors in parentheses.
Significance levels: $^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Cognition remains the primary driver of academic performance in all models. However, this expanded analysis reveals subtle gender differences, with boys exhibiting marginally higher cognitive elasticities in both subjects.

The inclusion of two distinct noncognitive factors offers a deeper understanding. While their impact is less pronounced than cognition, these factors often show statistical significance, particularly when assessed using the SDQ scale. Notably, girls demonstrate stronger noncognitive effects, especially in Maths performance, challenging simplistic gender-based assumptions about academic strengths.

Subject-wise comparisons indicate that cognition exerts a more substantial influence on Maths than on English across all models. Noncognitive factors, particularly for girls, play a more prominent role in Maths than might be expected.

The analysis highlights the superiority of the SDQ measures over TIPI in predicting academic outcomes, suggesting that targeted behavioral assessments may provide more accurate insights than broad personality traits in educational contexts. The examination of marginal products reinforces cognition's dominant role while also shedding light at the non-trivial contributions of noncognitive factors. The observed decreasing returns to scale, more pronounced in English, imply that proportional increases in all inputs yield diminishing academic gains.

C. Cobb-Douglas with two inputs

The Cobb-Douglas production function with two inputs is given by:

$$Y = AC^{\alpha}N^{\beta} \tag{8.32}$$

Where:

Y : Total output/Grade function/Academic achievement

A : Total factor productivity or scaling factor

C : Input representing cognition

N : Input representing noncognitive measures

α, β : Exponents determining the output response to each input

This function assumes a Cobb-Douglas form, where the exponents α and β indicate the elasticity or the degree of influence of each input on the total output Y. I test a three-input Cobb-Douglas in the Appendix. The similarity in parameters between this two-input model and the three-input model presented previously suggests that the combined effect of the chosen noncognitive variables Focused Behaviour and Conscientiousness effectively captures the majority of noncognitive influence on academic performance. This validates the use of a simplified model without significant loss of explanatory power.

1. Marginal products (MPs)

$$f_C = \left.\frac{\partial Y}{\partial C}\right|_{N=N_0} = A\alpha C^{\alpha-1}(N_0)^{\beta} = A\alpha\frac{C^{\alpha}N^{\beta}}{C} = \alpha\frac{Y}{C} \tag{8.33}$$

$$f_N = \left.\frac{\partial Y}{N}\right|_{C=C_0} = A\beta C_0^{\alpha}(N)^{\beta-1} = A\beta\frac{C^{\alpha}N^{\beta}}{N} = \beta\frac{Y}{N} \tag{8.34}$$

Taking the derivative of the marginal product of capital wrt capital (i.e., taking the second derivative of the production function wrt capital), we have:

$$\frac{\partial f_C}{\partial C} = A\alpha(\alpha-1)C^{\alpha-2}N^{\beta} = \alpha(\alpha-1)\frac{Y}{C^2} \tag{8.35}$$

$$\frac{\partial f_N}{\partial N} = A\beta(\beta-1)N^{\beta-2}C^{\alpha} = \beta(\beta-1)\frac{Y}{N^2} \tag{8.36}$$

2. Output elasticities (OEs)

Parameters estimation:

$$\ln(Y) = \ln(A) + \alpha \ln(C) + \beta \ln(N) \tag{8.37}$$

With output elasticities defined as:

$$OE_C = \left.\frac{\partial \ln(Y)}{\partial \ln(C)}\right|_{N=N_0} = \alpha \tag{8.38}$$

$$OE_N = \left.\frac{\partial \ln(Y)}{\partial \ln(N)}\right|_{C=C_0} = \beta \tag{8.39}$$

If we define the scale elasticity (SCE) as the scale change, often measured by the percent change in output from a simultaneous 1% change in all inputs, then: $OE_C + OE_N = \alpha + \beta$.

3. Elasticity of Substitution (ES)

$$\sigma = \frac{\frac{d(C/N)}{(C/N)}}{\frac{d(MP_C/MP_N)}{(MP_C/MP_N)}} \tag{8.40}$$

Where $MP_C(f_C)$ and $MP_N(f_N)$ are the marginal products of C and N respectively. For the Cobb-Douglas production function, the σ between the two inputs is always equal to 1. This is a key property of the Cobb-Douglas function. To illustrate this for the two-input case:

$$Y = AC^{\alpha}N^{1-\alpha} \tag{8.41}$$

The marginal products are:

$$f_C = \frac{\partial Y}{\partial C} = \alpha A C^{\alpha-1}N^{1-\alpha} \tag{8.42}$$

$$f_N = \frac{\partial Y}{\partial N} = (1-\alpha)AC^{\alpha}N^{-\alpha} \tag{8.43}$$

The ratio of marginal products is:

$$\frac{f_C}{f_N} = \frac{\alpha N}{(1-\alpha)C} \tag{8.44}$$

If we calculate σ, we find:

$$\sigma = \frac{\frac{d(C/N)}{(C/N)}}{\frac{d((\alpha N)/((1-\alpha)C))}{(\alpha N)/((1-\alpha)C)}} = 1 \tag{8.45}$$

This result of 1 holds for the two inputs in the Cobb-Douglas function, demonstrating the constant unitary elasticity of substitution between cognitive and noncognitive skills in this model.

4. Estimation and discussion

Table 9: Cobb-Douglas production function estimates for Maths achievement: comparison of TIPI and SDQ models across full sample and gender subgroups

	Maths			
Parameter	Full Sample	Boys	Girls	Δ (Boys - Girls)
TIPI Model				
A	0.256***	0.225***	0.264***	-0.039
	(0.015)	(0.019)	(0.021)	
α (Cognition)	0.781***	0.804***	0.774***	0.030
	(0.012)	(0.018)	(0.018)	
β (Conscientiousness)	0.024***	0.020***	0.023***	-0.003
	(0.003)	(0.004)	(0.004)	
Marginal Product (Cognition)	0.075	0.077	0.076	0.001
Marginal Product (Conscientiousness)	0.053	0.051	0.051	-0.001
Returns to Scale	0.805	0.830	0.800	0.030
SDQ Model				
A	0.274***	0.238***	0.274***	-0.036
	(0.018)	(0.025)	(0.024)	
α (Cognition)	0.742***	0.765***	0.729***	0.036
	(0.015)	(0.023)	(0.020)	
β (Focused Behaviour)	0.071***	0.053***	0.081***	-0.027
	(0.006)	(0.008)	(0.010)	
Marginal Product (Cognition)	0.072	0.073	0.072	0.002
Marginal Product (Focused Behaviour)	0.091	0.078	0.104	-0.027
Returns to Scale	0.814	0.828	0.819	0.010
Observations	5,631	2,667	2,801	

Standard errors in parentheses.
*Signif. Codes: ***: 0.001, **: 0.01, *: 0.05*

Note: The table displays estimates for the Cobb-Douglas production function applied to Maths scores using two inputs. The TIPI Model focuses on Cognition and Conscientiousness, while the SDQ Model considers Cognition and Focused Behaviour. Observations represent the number of data points for each group.

The cognitive factor α (OE_C) is the most significant predictor of academic performance across all models. For Maths (Table 9), boys show slightly higher cognitive output elasticities ($\alpha = 0.804$ for TIPI, 0.765 for SDQ) compared to girls ($\alpha = 0.774$ for TIPI, 0.729 for SDQ). This pattern is mirrored in English (Table 10), with boys' α ranging from 0.453 to 0.490 and girls' from 0.403 to 0.434. The full sample results fall between these gender-specific values, as expected.

The noncognitive factor β (OE_N) is smaller in magnitude but significant across all models, as expected. Focused Behaviour always shows stronger relationships than the Conscientiousness. For example, in the full sample Maths model, $\beta_{SDQ} = 0.071$ while $\beta_{TIPI} = 0.024$. This suggests that the SDQ may be a more sensitive measure of noncognitive skills relevant to academic performance.

Boys demonstrate a slightly, consistent, stronger cognitive component in both subjects.

Table 10: Cobb-Douglas production function estimates for English achievement: comparison of TIPI and SDQ models across full sample and gender subgroups

	English			
Parameter	Full Sample	Boys	Girls	Δ (Boys - Girls)
TIPI Model				
A	1.310***	1.020***	1.390***	-0.370
	(0.060)	(0.072)	(0.084)	
α (Cognition)	0.441***	0.490***	0.434***	0.056
	(0.010)	(0.015)	(0.013)	
β (Conscientiousness)	0.014***	0.009**	0.010**	-0.001
	(0.002)	(0.003)	(0.003)	
Marginal Product (Cognition)	0.045	0.048	0.046	0.003
Marginal Product (Conscientiousness)	0.032	0.025	0.023	0.002
Returns to Scale	0.454	0.502	0.445	0.057
SDQ Model				
A	1.400***	1.090***	1.430***	-0.340
	(0.079)	(0.107)	(0.105)	
α (Cognition)	0.405***	0.453***	0.403***	0.050
	(0.013)	(0.022)	(0.017)	
β (Focused Behaviour)	0.060***	0.044***	0.051***	-0.007
	(0.005)	(0.008)	(0.008)	
Marginal Product (Cognition)	0.041	0.045	0.043	0.002
Marginal Product (Focused Behaviour)	0.080	0.064	0.070	-0.005
Returns to Scale	0.465	0.502	0.458	0.044
Observations	5,631	2,667	2,801	

Standard errors in parentheses.
*Signif. Codes: ***: 0.001, **: 0.01, *: 0.05*

Note: The table displays estimates for the Cobb-Douglas production function applied to English scores using two inputs. The TIPI Model focuses on Cognition and Conscientiousness, while the SDQ Model considers Cognition and Focused Behaviour. Observations represent the number of data points for each group.

However, girls show stronger effects of noncognitive factors, particularly in Maths. This is especially evident with the SDQ measures, where girls' OE_N for Maths is 0.081 compared to boys' 0.053.

Both cognitive and noncognitive factors appear to have a stronger influence on Maths performance compared to English. This is evident in the higher values of both α and β for Maths across all models. For example, in the full sample SDQ model, $\alpha_{Maths} = 0.742$ while $\alpha_{English} = 0.405$. This suggests that the production function for academic achievement may vary across subjects, with Maths potentially being more sensitive to both cognitive and noncognitive inputs.

The marginal products further support these findings. Across all models, the marginal product for cognition is always higher than for noncognitive factors, reinforcing the dominant role of cognitive abilities in academic achievement.

The sum of α and β in all models is less than 1, which indicates decreasing returns to scale in the production of academic achievement. This implies that proportional increases in both cognitive and noncognitive inputs would result in less than proportional increases in academic output. For example, in the full sample Maths SDQ model, $\alpha + \beta = 0.742 + 0.071 = 0.813 < 1$. The returns to scale are always higher for Maths than for English, suggesting that the educational production process might be more efficient for Maths.

Overall, the two-input Cobb-Douglas model provides a simplified yet insightful perspective on the production of academic achievement.

D. Marginal Rate of Technical Substitution for the Cobb-Douglas production fuction with two and three inputs

The Marginal Rate of Technical Substitution (MRTS) is a concept in production theory that can be applied to the educational production function. In this context, the MRTS represents the rate at which one input (e.g., cognition) can be substituted for another (e.g., noncognitive skills) while maintaining the same level of output (academic performance). Mathematically, the MRTS is defined as the negative slope of the isoquant curve in input space.

In the Cobb-Douglas production function models, the MRTS helps us quantify the trade-offs between cognitive and noncognitive inputs in educational achievement. Specifically, we would be able to answer questions such as: how much improvement in noncognitive skills is needed to compensate for a deficit in cognitive abilities? To what extent can enhancements in one type of skill make up for deficiencies in another? Do these trade-offs differ between subjects (Maths vs. English) or between genders?

I will examine these questions using first the three two-input, employing the TIPI and SDQ measures for noncognitive skills (Conscientiousness and Emotional Stability for the TIPI, and Focused Behaviour and Emotional Resilience for the SDQ).

1. Definition - Three Inputs

We first start with a Cobb-Douglas production function with three-inputs:

$$Y = AC^{\alpha} N_E^{\beta_1} N_I^{\beta_2} \tag{8.46}$$

Given:

$$f_C = \frac{\partial Q}{\partial C} = \alpha AC^{\alpha-1} N_E^{\beta_1} N_I^{\beta_2} \tag{8.47}$$

$$f_{NE} = \frac{\partial Q}{\partial N_E} = \beta_1 AC^{\alpha} N_E^{\beta_1 - 1} N_I^{\beta_2} \tag{8.48}$$

$$f_{NI} = \frac{\partial Q}{\partial N_E} = \beta_2 AC^{\alpha} N_E^{\beta_1} N_I^{\beta_2 - 1} \tag{8.49}$$

Then MRTS:

$$MRTS_{N_E,C} = -\frac{dN_E}{dC} = \frac{f_C}{MP_{NE}} = \frac{\alpha AC^{\alpha-1} N_E^{\beta_1} N_I^{\beta_2}}{\beta_1 AC^{\alpha} N_E^{\beta_1 - 1} N_I^{\beta_2}} = \frac{\alpha}{\beta_1} \frac{N_E}{C} \tag{8.50}$$

$$MRTS_{N_I,C} = -\frac{dN_I}{dC} = \frac{f_C}{f_{NI}} = \frac{\alpha AC^{\alpha-1} N_E^{\beta_1} N_I^{\beta_2}}{\beta_2 AC^{\alpha} N_E^{\beta_1} N_I^{\beta_2 - 1}} = \frac{\alpha}{\beta_2} \frac{N_I}{C} \tag{8.51}$$

$$MRTS_{N_I,N_E} = -\frac{dN_I}{dN_E} = \frac{f_{NE}}{f_{NI}} = \frac{\beta_1 AC^{\alpha} N_E^{\beta_1 - 1} N_I^{\beta_2}}{\beta_2 AC^{\alpha} N_E^{\beta_1} N_I^{\beta_2 - 1}} = \frac{\beta_1}{\beta_2} \frac{N_I}{N_E} \tag{8.52}$$

2. Defition - Two Inputs

Followed the explanation from the MRTS with three-inputs in the previous section, we can now define the MRTS for a Cobb-Douglas with two inputs. Given:

$$Y = AC^{\alpha}N^{\beta} \tag{8.53}$$

Marginal products:

$$f_C = \frac{\partial Y}{\partial C}\bigg| N = N_0 = A\alpha C^{\alpha-1}(N_0)^{\beta} \tag{8.54}$$

$$f_N = \frac{\partial Y}{\partial N}\bigg| C = C_0 = A\beta C0^{\alpha}(N)^{\beta-1} \tag{8.55}$$

$$MRTS_{N,C} = \frac{f_C}{f_N} = \frac{A\alpha C^{\alpha-1}N^{\beta}}{A\beta C^{\alpha}N^{\beta-1}} = \frac{\alpha}{\beta}\frac{N}{C} \tag{8.56}$$

$MRTS_{N,C}$ is:

$$MRTS_{N,C} = \frac{\alpha}{\beta}\frac{N}{C} \tag{8.57}$$

$MRTS_{N,C}$ represents how much the noncognitive input (N) needs to increase to compensate for a unit decrease in cognition (C) while maintaining the same level of output (Y).

3. Estimation and Discussion

Table 11: Marginal Rates of Technical Substitution for 3-input Cobb-Douglas Models

		Marginal Rates of Technical Substitution (MRTS)			
Model	MRTS Type	Full Sample	Girls	Boys	Δ (Boys - Girls)
Maths (TIPI)	Emotional Stability for Cognition	3.600	3.619	2.724	-0.895
	Conscientiousness for Cognition	1.511	1.588	1.674	0.086
	Conscientiousness for Emo. Stability	0.420	0.439	0.615	0.176
Maths (SDQ)	Emotional Resilience for Cognition	2.599	2.678	2.064	-0.614
	Focused Behaviour for Cognition	0.824	0.723	1.018	0.295
	Focused Behaviour for Emo. Resilience	0.317	0.270	0.493	0.223
English (TIPI)	Emotional Stability for Cognition	5.427	7.357	3.190	-4.167
	Conscientiousness for Cognition	1.444	2.054	2.161	0.107
	Conscientiousness for Emo. Stability	0.266	0.279	0.677	0.398
English (SDQ)	Emotional Resilience for Cognition	14.013	3.163	2.279	-0.884
	Focused Behaviour for Cognition	0.520	0.636	0.734	0.098
	Focused Behaviour for Emo. Resilience	0.037	0.201	0.322	0.121

Note: The table presents the Marginal Rates of Technical Substitution Mathsfor 3-input Cobb-Douglas models in both Maths and English. The TIPI model uses Emotional Stability, Conscientiousness, and Cognition as inputs, while the SDQ model utilizes Emotional Resilience, Focused Behaviour, and Cognition. MRTS indicates the rate at which one input can be substituted for another while maintaining the same level of output. Δ represents the difference in MRTS between Boys and Girls.

Table 12: Marginal Rates of Technical Substitution for 2-input Cobb-Douglas models

		Marginal Rates of Technical Substitution (MRTS)			
Model	MRTS Type	Full Sample	Girls	Boys	Δ (Boys - Girls)
Maths (TIPI)	Conscientiousness for Cognition	1.420	1.475	1.520	0.045
	Cognition for Conscientiousness	0.704	0.678	0.658	-0.020
Maths (SDQ)	Focused Behaviour for Cognition	0.786	0.688	0.946	0.258
	Cognition for Focused Behaviour	1.272	1.454	1.058	-0.396
English (TIPI)	Conscientiousness for Cognition	1.390	1.966	1.950	-0.016
	Cognition for Conscientiousness	0.720	0.509	0.513	0.004
English (SDQ)	Focused Behaviour for Cognition	0.515	0.612	0.697	0.085
	Cognition for Focused Behaviour	1.941	1.634	1.434	-0.200

Note: The table presents the Marginal Rates of Technical Substitution Mathsfor 2-input Cobb-Douglas models in both Maths and English. The TIPI model uses Cognition and Conscientiousness as inputs, while the SDQ model utilizes Cognition and Focused Behaviour. MRTS is calculated as the ratio of the marginal product of one input to the marginal product of the other, indicating how inputs can be substituted while maintaining the same level of output.

In most cases, the MRTS of noncognitive skills for cognition is greater than 1, indicating that more than one unit of a noncognitive skill is needed to substitute for one unit of cognition. This further proves that cognition generally have a stronger impact on academic outcomes. For the TIPI variables, the MRTS of Conscientiousness for Cognition is higher in Maths than in English for the overall sample (1.420 vs 1.390). For the SDQ variables, the MRTS of Focused Behaviour for Cognition is higher in Maths than in English across all groups (0.786 vs 0.515 for the overall sample), which indicates that while Conscientiousness (TIPI) shows a relatively consistent connection to cognition across subjects, Focused Behaviour (SDQ) appears to have a stronger relative importance in Maths compared to English.

In Maths (TIPI), boys show a slightly higher MRTS of Conscientiousness for Cognition compared to girls (1.520 vs 1.475). In English (TIPI), both girls and boys show much higher MRTS of Conscientiousness for Cognition compared to the overall sample, with values close to 2 (1.966 for girls and 1.950 for boys). In Maths (SDQ), boys show a higher MRTS of Focused Behaviour for Cognition compared to girls (0.946 vs 0.688). This means that the trade-off between cognitive and noncognitive skills differs across genders, particularly in English performance.

The TIPI scale often shows higher MRTS values for noncognitive skills compared to the SDQ scale, particularly in English. When we consider the reciprocal MRTS (Cognition for Noncognitive skills), we see values less than 1 in most cases, particularly for English, meaning that cognition can more easily compensate for deficits in noncognitive skills than vice versa, especially in language performance.

We observe that for Maths, using the TIPI measure, 1.420 units of Conscientiousness are needed on average to compensate for 1 unit of Cognition, while for English, this ratio is slightly lower at 1.390. The SDQ measure presents a different picture: for Maths, 0.786 units of Focused Behaviour are required to substitute for 1 unit of Cognition, whereas in English, only 0.515 units are needed. Therefore, noncognitive skills, particularly as measured by the SDQ, play a more significant role relative to cognition in English compared to Maths. On the other hand, when we consider how cognitive skills can compensate for noncognitive skills, we find that for Maths (TIPI), 0.704 units of Cognition can substitute for 1 unit of Conscientiousness, while for English, this increases slightly to 0.720 units.

In English, we observe more dramatic variations in MRTS values compared to Maths, meaning that the relative importance of cognitive versus noncognitive skills is more subject to individual differences in language performance. This is particularly evident in the TIPI measure for English, where both girls and boys show MRTS values close to 2 (1.966 and 1.950, respectively) for Conscientiousness relative to Cognition, far higher than the overall sample average of 1.390. Gender differences are also apparent: girls demonstrate a higher variability in MRTS values between Maths and English compared to boys, suggesting that the relative importance of cognitive versus noncognitive skills changes more dramatically for girls across subjects. The SDQ measure (Focused Behaviour) shows a more consistent pattern across subjects and genders compared to the TIPI measure (Conscientiousness), potentially indicating that specific behavioral traits have a more uniform connection to academic performance across different contexts.

E. Derivations for a Translog production function with two inputs

$$Y = AC^{\alpha}N^{\beta}\exp\left\{\frac{1}{2}\gamma_1\left[\ln(C)\right]^2 + \frac{1}{2}\gamma_2\left[\ln(N)\right]^2 + \gamma_{12}\ln(C)\ln(N)\right\} \tag{8.58}$$

Where:

- Y is the output (educational achievement)
- A is the total factor productivity
- C and N are the inputs (Cognition and Noncognitive skills)
- α and β are the direct effects of inputs
- γ_1, γ_2, and γ_{12} capture the non-linear and interaction effects

1. Marginal Products (MPs)

$$f_C = \left.\frac{\partial Y}{\partial C}\right|_{N=N_0} = A\alpha C^{\alpha-1}N_0^{\beta}\frac{\partial}{\partial C}\left[\exp\left\{X(C,N_0)\right\}\right] \tag{8.59}$$

$$f_N = \left.\frac{\partial Y}{\partial N}\right|_{C=C_0} = AC_0^{\alpha}\beta N^{\beta-1}\frac{\partial}{\partial N}\left[\exp\left\{X(C_0,N)\right\}\right] \tag{8.60}$$

where $X(C,N)$ is the exponential term in the original function:

$$X(C,N) = \frac{1}{2}\gamma_1\left[\ln(C)\right]^2 + \frac{1}{2}\gamma_2\left[\ln(N)\right]^2 + \gamma_{12}\ln(C)\ln(N) \tag{8.61}$$

After applying the chain rule and simplifying, we get the full expressions for the marginal products:

$$f_C = A\alpha C^{\alpha-1}N_0^{\beta}\exp\left\{X(C,N_0)\right\}\left[1+\gamma_1\ln(C)+\gamma_{12}\ln(N_0)\right] \tag{8.62}$$

$$f_N = A\beta C_0^{\alpha}N^{\beta-1}\exp\left\{X(C_0,N)\right\}\left[1+\gamma_2\ln(N)+\gamma_{12}\ln(C_0)\right] \tag{8.63}$$

2. Output Elasticities (OEs)

$$\ln Y = \ln A + \alpha \ln C + \beta \ln N + \frac{1}{2}\gamma_1 (\ln C)^2 + \frac{1}{2}\gamma_2 (\ln N)^2 + \gamma_{12} \ln C \ln N \tag{8.64}$$

To derive the output elasticities, we take the partial derivatives of this equation with respect to $\ln(C)$ and $\ln(N)$:

For C:

$$OE_C = \left.\frac{\partial \ln(Y)}{\partial \ln(C)}\right|_{N=N_0} \tag{8.65}$$
$$= \frac{\partial}{\partial \ln(C)}\left[\ln A + \alpha \ln C + \beta \ln N_0 + \frac{1}{2}\gamma_1 (\ln C)^2 + \frac{1}{2}\gamma_2 (\ln N_0)^2 + \gamma_{12} \ln C \ln N_0\right] \tag{8.66}$$
$$= \alpha + \gamma_1 \ln(C) + \gamma_{12} \ln(N_0) \tag{8.67}$$

For N:

$$OE_N = \left.\frac{\partial \ln(Y)}{\partial \ln(N)}\right|_{C=C_0} \tag{8.68}$$
$$= \frac{\partial}{\partial \ln(N)}\left[\ln A + \alpha \ln C_0 + \beta \ln N + \frac{1}{2}\gamma_1 (\ln C_0)^2 + \frac{1}{2}\gamma_2 (\ln N)^2 + \gamma_{12} \ln C_0 \ln N\right] \tag{8.69}$$
$$= \beta + \gamma_2 \ln(N) + \gamma_{12} \ln(C_0) \tag{8.70}$$

Therefore, the output elasticities are:

$$OE_C = \left.\frac{\partial \ln(Y)}{\partial \ln(C)}\right|_{N=N_0} = \alpha + \gamma_1 \ln(C) + \gamma_{12} \ln(N_0) \tag{8.71}$$

$$OE_N = \left.\frac{\partial \ln(Y)}{\partial \ln(N)}\right|_{C=C_0} = \beta + \gamma_2 \ln(N) + \gamma_{12} \ln(C_0) \tag{8.72}$$

3. Advantages

The Translog model provides a more flexible functional form compared to the Cobb-Douglas models discussed in previous sections, which allows us to capture non-linear relationships and interactions between cognitive and noncognitive inputs that were not possible in simpler specifications. Some of the key-features of these output elasticities are: variable elasticities, input interactions, and non-linearity. Unlike in the Cobb-Douglas model where elasticities are constant, in the Translog model, elasticities vary with the levels of inputs. The term $\gamma_{12}\ln(N_0)$ in OE_C and $\gamma_{12}\ln(C_0)$ in OE_N capture how the elasticity of one input depends on the level of the other input. The terms $\gamma_1\ln(C)$ and $\gamma_2\ln(N)$ allow for non-linear relationships between inputs and output.

These features make the Translog function useful for modeling non-obvious educational production processes where the impacts of cognitive and noncognitive skills may vary at different levels and interact with each other.

4. Elasticity of Substitution (ES)

$$\sigma = 1 - \frac{\partial \ln(MRTS)}{\partial \ln(C/N)} \tag{8.73}$$

2) We know that $MRTS = \frac{f_C}{f_N} = \frac{OE_C}{OE_N} \cdot \frac{N}{C}$, so:

$$\ln(MRTS) = \ln(OE_C) - \ln(OE_N) + \ln(N) - \ln(C) \tag{8.74}$$

3) We need to find $\frac{\partial \ln(MRTS)}{\partial \ln(C/N)}$. First, let us express $\ln(C/N)$:

$$\ln(C/N) = \ln(C) - \ln(N) \tag{8.75}$$

4) Now, we can calculate $\frac{\partial \ln(MRTS)}{\partial \ln(C/N)}$:

$$\frac{\partial \ln(MRTS)}{\partial \ln(C/N)} = \frac{\partial \ln(OE_C)}{\partial \ln(C)} - \frac{\partial \ln(OE_N)}{\partial \ln(C)} - \frac{\partial \ln(OE_C)}{\partial \ln(N)} + \frac{\partial \ln(OE_N)}{\partial \ln(N)} - 1 \tag{8.76}$$

5) Using the output elasticities we derived earlier:
$OE_C = \alpha + \gamma_1 \ln(C) + \gamma_{12} \ln(N)$ $OE_N = \beta + \gamma_2 \ln(N) + \gamma_{12} \ln(C)$
We can calculate:
$\frac{\partial \ln(OE_C)}{\partial \ln(C)} = \frac{\gamma_1}{OE_C}$ $\frac{\partial \ln(OE_N)}{\partial \ln(C)} = \frac{\gamma_{12}}{OE_N}$ $\frac{\partial \ln(OE_C)}{\partial \ln(N)} = \frac{\gamma_{12}}{OE_C}$ $\frac{\partial \ln(OE_N)}{\partial \ln(N)} = \frac{\gamma_2}{OE_N}$
6) Substituting these back into the equation:

$$\frac{\partial \ln(MRTS)}{\partial \ln(C/N)} = \frac{\gamma_1}{OE_C} - \frac{\gamma_{12}}{OE_N} - \frac{\gamma_{12}}{OE_C} + \frac{\gamma_2}{OE_N} - 1 \tag{8.77}$$

7) Finally, substituting this into the original formula for σ:

$$\sigma = 1 - \left(\frac{\gamma_1}{OE_C} - \frac{\gamma_{12}}{OE_N} - \frac{\gamma_{12}}{OE_C} + \frac{\gamma_2}{OE_N} - 1 \right) \tag{8.78}$$

8) Simplifying:

$$\sigma = 2 - \frac{\gamma_1}{OE_C} + \frac{\gamma_{12}}{OE_N} + \frac{\gamma_{12}}{OE_C} - \frac{\gamma_2}{OE_N} \tag{8.79}$$

5. Marginal Rate of Technical Substitution (MRTS)

1) By definition, $MRTS_{CN} = \frac{f_C}{f_N}$
2) We know that $f_C = OE_C \cdot \frac{Y}{C}$ and $f_N = OE_N \cdot \frac{Y}{N}$
3) Therefore:

$$MRTS_{CN} = \frac{f_C}{f_N} = \frac{OE_C \cdot \frac{Y}{C}}{OE_N \cdot \frac{Y}{N}} = \frac{OE_C}{OE_N} \cdot \frac{N}{C} \tag{8.80}$$

4) Substituting the expressions for OE_C and OE_N:

$$MRTS_{CN} = \frac{\alpha + \gamma_1 \ln(C) + \gamma_{12} \ln(N)}{\beta + \gamma_2 \ln(N) + \gamma_{12} \ln(C)} \cdot \frac{N}{C} \tag{8.81}$$

F. General form: CES with three inputs

$$Y = A\left[\alpha C^{\rho} + (1-\alpha)\left(\beta N_E^{\rho} + (1-\beta)N_I^{\rho}\right)\right]^{\frac{1}{\rho}} \tag{8.82}$$

Where:

$$\begin{aligned}
Y &: \text{Total output/Grade function/Academic achievement} \\
A &: \text{Total factor productivity or scaling factor} \\
\alpha &: \text{Share parameter for cognitive input} \\
\beta &: \text{Share parameter for noncognitive inputs} \\
\rho &: \text{Substitution parameter, where } \rho = \frac{\sigma - 1}{\sigma} \\
\sigma &: \text{Elasticity of substitution} \\
C &: \text{Input representing cognition} \\
N_E, N_I &: \text{Inputs representing noncognitive measures}
\end{aligned}$$

This function represents a Constant Elasticity of Substitution (CES) production function with three inputs. The CES function is more flexible than the Cobb-Douglas form, allowing for varying degrees of substitutability between inputs. The elasticity of substitution (σ) between any pair of inputs is constant and determined by the parameter ρ.

In this model, C represents a measure of cognition (in this case, the principal component as a composite of three cognitive measures). N_E and N_I represent noncognitive measures, which I call External Control and Internal Control, respectively. In relation to the scales used (TIPI and SDQ), Internal Control serves as a proxy for Focused behaviour and Conscientiousness, while External Control proxies for Emotional Resilience and Emotional Stability. These four variables appear to be the most significant based on my analysis.

The share parameters α and β determine the relative importance of the inputs in the production function. However, unlike in a Cobb-Douglas function, they do not directly represent output elasticities. In a CES function, the output elasticities are variable and depend on the levels of inputs used.

1. Elasticity of Substitution

The elasticity of substitution (σ) in the three-input CES model measures the ease of substitution between any pair of inputs while holding the third input constant. The relationship between σ and ρ is:

- When $\sigma > 1$ (or $-1 < \rho < \infty$), any pair of inputs are substitutes.
- When $\sigma < 1$ (or $\rho < -1$), any pair of inputs are complements.
- As σ approaches infinity (or ρ approaches 1), the inputs become perfect substitutes.
- As σ approaches 0 (or ρ approaches $-\infty$), the inputs become perfect complements.
- When $\sigma = 1$ (or $\rho = 0$), the CES function reduces to the Cobb-Douglas form.

In the context of educational production with cognitive (C), external noncognitive (N_E), and internal noncognitive (N_I) inputs, these relationships indicate how these different skills interact in producing educational outcomes. For example:

- If $\sigma > 1$, it suggests that a deficiency in one type of skill (e.g., cognitive) can be more easily compensated by either of the other skills (external or internal noncognitive).
- If $\sigma < 1$, it implies that all three types of skills are complementary, and a balanced development of all skills is important for educational outcomes.

While σ provides a measure of overall substitutability, the specific substitution relationships between the pairs of inputs (C and N_E, C and N_I, N_E and N_I) may vary. In the proposed model I assume a constant elasticity of substitution between all pairs of inputs, which is a simplification of potentially non-linear relationships in real life.

2. Marginal products (MPs)

$$f_C = \left.\frac{\partial Y}{\partial C}\right|_{N_E=N_{E0},N_I=N_{I0}} = A\alpha C^{\rho-1}\left[\alpha C^{\rho}+(1-\alpha)\left(\beta N_{E0}^{\rho}+(1-\beta)N_{I0}^{\rho}\right)\right]^{\frac{1}{\rho}-1} \tag{8.83}$$

$$f_{NE} = \left.\frac{\partial Y}{\partial N_E}\right|_{C=C_0,N_I=N_{I0}} = A(1-\alpha)\beta N_E^{\rho-1}\left[\alpha C_0^{\rho}+(1-\alpha)\left(\beta N_E^{\rho}+(1-\beta)N_{I0}^{\rho}\right)\right]^{\frac{1}{\rho}-1} \tag{8.84}$$

$$f_{NI} = \left.\frac{\partial Y}{\partial N_I}\right|_{C=C_0,N_E=N_{E0}} = A(1-\alpha)(1-\beta)N_I^{\rho-1}\left[\alpha C_0^{\rho}+(1-\alpha)\left(\beta N_{E0}^{\rho}+(1-\beta)N_I^{\rho}\right)\right]^{\frac{1}{\rho}-1} \tag{8.85}$$

3. Output elasticities (OEs)

Given:

$$Y = A\left[\alpha C^{\rho}+(1-\alpha)\left(\beta N_E^{\rho}+(1-\beta)N_I^{\rho}\right)\right]^{\frac{1}{\rho}} \tag{8.86}$$

When we take the log on both sides:

$$\ln(Y) = \ln(A)+\frac{1}{\rho}\ln\left[\alpha C^{\rho}+(1-\alpha)\left(\beta N_E^{\rho}+(1-\beta)N_I^{\rho}\right)\right] \tag{8.87}$$

We find the output elasticities for C, OE_{N_E} and OE_{N_I}:

$$OE_C = \left.\frac{\partial \ln(Y)}{\partial \ln(C)}\right|_{N_E=N_{E0},N_I=N_{I0}} = \frac{\alpha C^{\rho}}{\alpha C^{\rho}+(1-\alpha)\left(\beta N_{E0}^{\rho}+(1-\beta)N_{I0}^{\rho}\right)} \tag{8.88}$$

$$OE_{N_E} = \left.\frac{\partial \ln(Y)}{\partial \ln(N_E)}\right|_{C=C_0,N_I=N_{I0}} = \frac{(1-\alpha)\beta N_E^{\rho}}{\alpha C_0^{\rho}+(1-\alpha)\left(\beta N_E^{\rho}+(1-\beta)N_{I0}^{\rho}\right)} \tag{8.89}$$

$$OE_{N_I} = \left.\frac{\partial \ln(Y)}{\partial \ln(N_I)}\right|_{C=C_0,N_E=N_{E0}} = \frac{(1-\alpha)(1-\beta)N_I^{\rho}}{\alpha C_0^{\rho}+(1-\alpha)\left(\beta N_{E0}^{\rho}+(1-\beta)N_I^{\rho}\right)} \tag{8.90}$$

Step-by-step derivation:
1) First we take the natural logarithm:

$$\ln(Y) = \ln(A)+\frac{1}{\rho}\ln\left[\alpha C^{\rho}+(1-\alpha)\left(\beta N_E^{\rho}+(1-\beta)N_I^{\rho}\right)\right] \tag{8.91}$$

2) Then we derive the output elasticities one by one:

For Cognitive input (C):

$$OE_C = \frac{\partial \ln(Y)}{\partial \ln(C)} = \frac{\alpha C^\rho}{\alpha C^\rho + (1-\alpha)\left(\beta N_E^\rho + (1-\beta)N_I^\rho\right)} \tag{8.92}$$

For External Noncognitive input (N_E):

$$OE_{N_E} = \frac{\partial \ln(Y)}{\partial \ln(N_E)} = \frac{(1-\alpha)\beta N_E^\rho}{\alpha C^\rho + (1-\alpha)\left(\beta N_E^\rho + (1-\beta)N_I^\rho\right)} \tag{8.93}$$

For Internal Noncognitive input (N_I):

$$OE_{N_I} = \frac{\partial \ln(Y)}{\partial \ln(N_I)} = \frac{(1-\alpha)(1-\beta)N_I^\rho}{\alpha C^\rho + (1-\alpha)\left(\beta N_E^\rho + (1-\beta)N_I^\rho\right)} \tag{8.94}$$

4. Returns to scale

The degree of returns to scale is determined by the sum of all output elasticities. We can call this sum the scale elasticity (SE):

$$SE = OE_C + OE_{N_E} + OE_{N_I} \tag{8.95}$$

Then:
a) If $SE > 1$ = Increasing returns to scale;
b) If $SE < 1$ = Decreasing returns to scale;
c) If $SE = 1$ = Constant returns to scale.
For:

$$Y = A\left[\alpha C^\rho + (1-\alpha)\left(\beta N_E^\rho + (1-\beta)N_I^\rho\right)\right]^{\frac{1}{\rho}} \tag{8.96}$$

The sum of the output elasticities is always 1, regardless of the parameter values:

$$OE_C + OE_{N_E} + OE_{N_I} = \frac{\alpha C^\rho + (1-\alpha)\beta N_E^\rho + (1-\alpha)(1-\beta)N_I^\rho}{\alpha C^\rho + (1-\alpha)\left(\beta N_E^\rho + (1-\beta)N_I^\rho\right)} = 1 \tag{8.97}$$

This 3-input CES function exhibits constant returns to scale by construction. This is a property of the CES function with the exponent $\frac{1}{\rho}$ outside the brackets. In the context of cognition and noncognition as inputs in an educational production function, it means that if we increase all three inputs by a factor k then output Y also increases by factor k. More specifically:

1. Proportional increase in inputs:

$$C \to kC, \quad N_E \to kN_E, \quad N_I \to kN_I \tag{8.98}$$

2. Resulting increase in output:

$$Y(kC, kN_E, kN_I) = kY(C, N_E, N_I) \tag{8.99}$$

In practical terms for education, this means a proportional improvement in cognitive and noncognitive skills leads to an equivalent proportional improvement in educational outcomes. For example, if we could somehow double (k = 2) a student's cognitive ability (C) and both types of noncognitive abilities (N_E and N_I) simultaneously, we would expect their educational output (Y, measured by test scores as a proxy for overall academic performance) to also double. This implies:

a) No diminishing returns when scaling up all inputs equally;

b) No extra benefits (increasing returns) when scaling up all inputs equally.

We need to keep in mind that this is a simplification of a sophisticated reality. In practice, the links between cognitive abilities, noncognitive skills, and educational outcomes is more nuanced and also most-likely non-linear, as we have seen in previous chapters.

If we were to allow for different returns to scale, we could modify the CES function to:

$$Y = A\left[\alpha C^{\rho} + (1-\alpha)\left(\beta N_E^{\rho} + (1-\beta)N_I^{\rho}\right)\right]^{\frac{\nu}{\rho}} \tag{8.100}$$

Where ν is a new parameter that determines the overall returns to scale:

a) If $\nu > 1$ = Increasing returns to scale;

b) If $\nu < 1$ = Decreasing returns to scale;

c) If $\nu = 1$ = Constant returns to scale (current case).

5. Marginal Rate of Technical Substitution for Three-Input CES

For the three-input CES production function:

$$Q = A\left[\alpha C^{\rho} + (1-\alpha)\left(\beta N_E^{\rho} + (1-\beta)N_I^{\rho}\right)\right]^{\frac{1}{\rho}} \tag{8.101}$$

The marginal products are:

$$f_C = \frac{\partial Q}{\partial C} = A\alpha C^{\rho-1}\left[\alpha C^{\rho} + (1-\alpha)\left(\beta N_E^{\rho} + (1-\beta)N_I^{\rho}\right)\right]^{\frac{1}{\rho}-1} \tag{8.102}$$

$$f_{N_E} = \frac{\partial Q}{\partial N_E} = A(1-\alpha)\beta N_E^{\rho-1}\left[\alpha C^{\rho} + (1-\alpha)\left(\beta N_E^{\rho} + (1-\beta)N_I^{\rho}\right)\right]^{\frac{1}{\rho}-1} \tag{8.103}$$

$$f_{N_I} = \frac{\partial Q}{\partial N_I} = A(1-\alpha)(1-\beta)N_I^{\rho-1}\left[\alpha C^{\rho} + (1-\alpha)\left(\beta N_E^{\rho} + (1-\beta)N_I^{\rho}\right)\right]^{\frac{1}{\rho}-1} \tag{8.104}$$

The MRTS can be calculated for each pair of inputs:

1. Between cognitive (C) and external noncognitive (N_E) inputs:

$$MRTS_{C,N_E} = \frac{f_C}{f_{N_E}} = \frac{\alpha}{\beta(1-\alpha)}\left(\frac{N_E}{C}\right)^{1-\rho} \tag{8.105}$$

2. Between cognitive (C) and internal noncognitive (N_I) inputs:

$$MRTS_{C,N_I} = \frac{f_C}{f_{N_I}} = \frac{\alpha}{(1-\beta)(1-\alpha)}\left(\frac{N_I}{C}\right)^{1-\rho} \tag{8.106}$$

3. Between external noncognitive (N_E) and internal noncognitive (N_I) inputs:

$$MRTS_{N_E,N_I} = \frac{f_{N_E}}{f_{N_I}} = \frac{\beta}{1-\beta}\left(\frac{N_I}{N_E}\right)^{1-\rho} \tag{8.107}$$

These MRTS formulas demonstrate how the substitutability between each pair of inputs changes with their relative quantities and the elasticity of substitution parameter ρ. We can analyze the trade-offs between any two of the three inputs while holding the third constant.

For example, $MRTS_{C,N_E}$ shows how much external noncognitive input (N_E) is needed to compensate for a small decrease in cognitive input (C) while maintaining the same output level and holding internal noncognitive input (N_I) constant. The relative ease of this substitution is influenced by the parameters α, β, and ρ, as well as the current levels of C and N_E.

6. Isoquants

Isoquants (iso = same, quant = quantity) for the three-input CES production function represent combinations of C, N_E, and N_I that produce the same level of output Y. Due to the three-dimensional nature of the input space, we can represent isoquants in a few ways:

1. Two-dimensional representation:

   Fixing C at a level C_0, we can represent the isoquant for output level Y_0 as:

   $$N_I = \left[\frac{(Y_0^\rho/A^\rho - \alpha C_0^\rho)}{(1-\alpha)} - \beta N_E^\rho\right]^{1/\rho} /(1-\beta)^{1/\rho} \tag{8.108}$$

   This equation gives combinations of N_E and N_I that produce output Y_0 when $C = C_0$.

2. Three-dimensional representation:

   The full isoquant surface for output level Y_0 is given by:

   $$Y_0 = A\left[\alpha C^\rho + (1-\alpha)(\beta N_E^\rho + (1-\beta)N_I^\rho)\right]^{1/\rho} \tag{8.109}$$

   This surface in (C, N_E, N_I) space represents all combinations of inputs producing output Y_0.

The shape of the isoquants reflects the substitutability between inputs. As ρ approaches 1 (perfect substitutes), the isoquants become more linear. As ρ approaches negative infinity (perfect complements), the isoquants approach right angles.

G. General form: CES with two inputs

$$Y = A\left[\alpha C^\rho + (1-\alpha)N^\rho\right]^{\frac{1}{\rho}} \tag{8.110}$$

Where:

Y : Total output/Grade function/Academic achievement
A : Total factor productivity or scaling factor
α : Share parameter for cognitive input
ρ : Substitution parameter, where $\rho = \dfrac{\sigma-1}{\sigma}$
σ : Elasticity of substitution
C : Input representing cognition
N : Input representing noncognitive measure

1. Elasticity of Substitution

The elasticity of substitution (σ) measures how easily cognitive and noncognitive inputs can be substituted for each other. Its connection to ρ is important for understanding the nature of input substitutability:

- When $\sigma > 1$ (or $-1 < \rho < \infty$), cognitive and noncognitive inputs are substitutes.

- When $\sigma < 1$ (or $\rho < -1$), cognitive and noncognitive inputs are complements.
- As σ approaches infinity (or ρ approaches 1), the inputs become perfect substitutes.
- As σ approaches 0 (or ρ approaches $-\infty$), the inputs become perfect complements.
- When $\sigma = 1$ (or $\rho = 0$), the CES function reduces to the Cobb-Douglas form.

In the context of educational production, these links indicate how cognitive and noncognitive skills interact in producing educational outcomes. For example, if $\sigma > 1$, it suggests that a deficiency in one type of skill can be more easily compensated by the other.

2. Marginal Products (MPs)

$$f_C = \frac{\partial Y}{\partial C} = A\alpha C^{\rho-1}\left[\alpha C^{\rho} + (1-\alpha)N^{\rho}\right]^{\frac{1}{\rho}-1} \tag{8.111}$$

$$f_N = \frac{\partial Y}{\partial N} = A(1-\alpha)N^{\rho-1}\left[\alpha C^{\rho} + (1-\alpha)N^{\rho}\right]^{\frac{1}{\rho}-1} \tag{8.112}$$

3. Output elasticities (OEs)

$$OE_C = \frac{\partial \ln(Y)}{\partial \ln(C)} = \frac{\alpha C^{\rho}}{\alpha C^{\rho} + (1-\alpha)N^{\rho}} \tag{8.113}$$

$$OE_N = \frac{\partial \ln(Y)}{\partial \ln(N)} = \frac{(1-\alpha)N^{\rho}}{\alpha C^{\rho} + (1-\alpha)N^{\rho}} \tag{8.114}$$

The sum of output elasticities would still be 1, indicating constant returns to scale:

$$OE_C + OE_N = \frac{\alpha C^{\rho} + (1-\alpha)N^{\rho}}{\alpha C^{\rho} + (1-\alpha)N^{\rho}} = 1 \tag{8.115}$$

4. Returns to scale

The interpretation of constant returns to scale remains the same as in the three-input case: a proportional increase in both cognitive and noncognitive inputs leads to an equivalent proportional increase in the educational output.

The degree of returns to scale is determined by the sum of all output elasticities. We can call this sum the scale elasticity (SE):

$$SE = OE_C + OE_N \tag{8.116}$$

Then:
a) If $SE > 1$ = Increasing returns to scale;
b) If $SE < 1$ = Decreasing returns to scale;
c) If $SE = 1$ = Constant returns to scale.
For:

$$Y = A\left[\alpha C^{\rho} + (1-\alpha)N^{\rho}\right]^{\frac{1}{\rho}} \tag{8.117}$$

The sum of the output elasticities is always 1, regardless of the parameter values:

$$OE_C + OE_N = \frac{\alpha C^{\rho} + (1-\alpha)N^{\rho}}{\alpha C^{\rho} + (1-\alpha)N^{\rho}} = 1 \tag{8.118}$$

the 2-input CES function exhibits constant returns to scale by construction. This is a property of the CES function with the exponent $\frac{1}{\rho}$ outside the brackets. In the context of cognition and noncognition as inputs in an educational production function, it means that if we increase all three inputs by a factor k then output Y also increases by factor k. More specifically:

1. Proportional increase in inputs:

$$C \to kC, \quad N \to kN \tag{8.119}$$

2. Resulting increase in output:

$$Y(kC, kN) = kY(C, N) \tag{8.120}$$

In practical terms for education, this means a proportional improvement in cognitive and noncognitive skills leads to an equivalent proportional improvement in educational outcomes.

If we were to allow for different returns to scale, we could modify the CES function to:

$$Y = A\left[\alpha C^{\rho} + (1-\alpha)N^{\rho}\right]^{\frac{\nu}{\rho}} \tag{8.121}$$

Where ν is a new parameter that determines the overall returns to scale:
a) If $\nu > 1$ = Increasing returns to scale;
b) If $\nu < 1$ = Decreasing returns to scale;
c) If $\nu = 1$ = Constant returns to scale (current case).

5. Marginal Rate of Technical Substitution (MRTS)

For the two-input CES production function:

$$Q = A\left(\alpha C^{\rho} + (1-\alpha)N^{\rho}\right)^{\frac{1}{\rho}} \tag{8.122}$$

The marginal products are:

$$f_C = \frac{\partial Q}{\partial C} = A\alpha\left(\alpha C^{\rho} + (1-\alpha)N^{\rho}\right)^{\frac{1}{\rho}-1} C^{\rho-1} \tag{8.123}$$

$$f_N = \frac{\partial Q}{\partial N} = A(1-\alpha)\left(\alpha C^{\rho} + (1-\alpha)N^{\rho}\right)^{\frac{1}{\rho}-1} N^{\rho-1} \tag{8.124}$$

$$MRTS_{CN} = \frac{f_C}{f_N} = \frac{\alpha C^{\rho-1}}{(1-\alpha)N^{\rho-1}} = \frac{\alpha}{1-\alpha}\left(\frac{C}{N}\right)^{\rho-1} \tag{8.125}$$

6. Isoquants

Isoquants for the two-input CES production function represent combinations of C and N that produce the same level of output Y. For the two-input case, we can represent isoquants as follows:

1. Equation form:

   For a given output level Y_0, the isoquant is represented by:

$$Y_0 = A\left[\alpha C^{\rho} + (1-\alpha)N^{\rho}\right]^{1/\rho} \tag{8.126}$$

This can be rearranged to express N in terms of C:

$$N = \left[\frac{(Y_0^\rho / A^\rho) - \alpha C^\rho}{1 - \alpha} \right]^{1/\rho} \tag{8.127}$$

2. Graphical representation:

   In the two-dimensional space of C and N, each isoquant is a curve representing all combinations of cognitive and noncognitive inputs that produce the same level of output Y_0.

The shape of the isoquants reflects the substitutability between cognitive and noncognitive inputs:

- As ρ approaches 1 (or σ approaches infinity), the isoquants become more linear, indicating that C and N are close to perfect substitutes.
- As ρ approaches negative infinity (or σ approaches 0), the isoquants approach right angles, indicating that C and N are close to perfect complements.
- When $\rho = 0$ (or $\sigma = 1$), the isoquants take on the familiar Cobb-Douglas shape.

These isoquant properties offer an understanding of how cognitive and noncognitive skills can be combined to achieve the same educational outcome, and how this combination varies with the elasticity of substitution between these skills.

H. Limitations

While the two-input CES model further enhances our understanding of the connection between cognitive and noncognitive skills in educational production, I have to note it has some limitations. Talking about the selection of noncognitive skills, the proposed model only considers one noncognitive skill as a single input, which may oversimplify the nature of noncognition. The model also assumes a constant elasticity of substitution, which may not always hold across different levels of inputs. Other factors that influence educational outcomes, such as family background or school quality, are not explicitly included in this model (but are in the regressions).

I. Comparison between two-input and three-input CES models

The choice between the two-input and three-input CES models involves several trade-offs and was put on hold due to several empirical (mainly computational) limitations:

Simplicity vs. complexity: The two-input model offers greater simplicity and it is easier to interpret, which makes it more manageable for theoretical analysis and easier to estimate empirically. However, the three-input model allows for a more nuanced representation of noncognitive skills (distinguishing between internal and external control) and allows us to choose the inputs which seem to be more relevant. If we were to have better measures that use the same scale it would be easier to draw cross-model comparisons. The fact that both the TIPI and the SDQ scales measure different aspects of noncognition constrains us from running principal-component analysis.

Parsimony vs. granularity: The two-input model is more ”parsimonious”, requiring fewer parameters (just one for the cognitive input and one for the noncognitive) to be estimated.

This can be an advantage when working with limited data, especially concerning measures of noncognition. The three-input model, while requiring more parameters, provides a more granular view of the production process. They both have their pros and cons, and researcher discretion is advised.

Generalizability vs. specificity: The two-input model may be more generalizable across different contexts where noncognitive skills are not easily separable. The three-input model is more specific to contexts where distinct aspects of noncognitive skills (like internal and external control) are more "straight-foward" identifiable and measurable.

The choice between these models really depends on the research question, data availability, and the specific educational context being studied (not to mention the patience to try different optimization mechanisms when choosing the starting values for the parameters).

1. Nested CES function

While the previous analysis focuses on the standard CES function, it is worth noting the possibility of using a nested CES function, particularly for the three-input case. The first specification was:

$$Y = A\left[\alpha C^{\rho} + (1-\alpha)\left(\beta N_E^{\rho} + (1-\beta)N_I^{\rho}\right)\right]^{\frac{1}{\rho}} \tag{8.128}$$

While a nested CES function could look like:

$$Y = A\left[\alpha C^{\rho} + (1-\alpha)\left(\beta N_E^{\gamma} + (1-\beta)N_I^{\gamma}\right)^{\rho/\gamma}\right]^{\frac{1}{\rho}} \tag{8.129}$$

This nested specification allows for different elasticities of substitution between cognitive and noncognitive inputs (determined by ρ) and between the two types of noncognitive inputs (determined by γ).

This additional parameter, γ, allows for different elasticities of substitution between inputs:

- ρ determines the elasticity of substitution between cognitive skills (C) and the composite of noncognitive skills (N_E and N_I).
- γ determines the elasticity of substitution between the two types of noncognitive skills (N_E and N_I).
- When $\gamma > \rho$, the two noncognitive inputs (N_E and N_I) are more substitutable with each other than either is with the cognitive input (C).
- When $\gamma < \rho$, the noncognitive inputs are less substitutable with each other than with the cognitive input.
- When $\gamma = \rho$, the nested CES function reduces to the standard three-input CES function.

This nested structure is more interesting at first because it allows for detailed modeling of the links between different types of skills. For example, it can capture scenarios where external and internal noncognitive skills might be more easily substituted for each other than either can be for cognitive skills.

In educational terms, a high γ relative to ρ might suggest that deficiencies in one type of noncognitive skill (e.g., external control) can be more easily compensated by strengths in the other noncognitive skill (e.g., internal control) than by cognitive abilities.

However, while this alternative model could capture more nuanced links between the inputs, it would be even more computationally complex to estimate, and the researcher would have to justify the choice of variables.

2. Economic interpretation of parameters

The parameters in the previous CES models have important economic interpretations in the context of educational production:

- α (and β in the three-input case) represent the relative importance of different inputs. For example, a higher α suggests that cognitive skills contribute more to educational outcomes relative to noncognitive skills. These parameters can be interpreted as technology parameters that reflect the current state of the educational production process.

- ρ (or equivalently, σ) indicates the degree of substitutability between inputs. In educational terms, this reflects how easily deficiencies in one type of skill can be compensated by strengths in another. A higher σ suggests greater flexibility in combining different skills to achieve educational outcomes.

- A represents total factor productivity, which in an educational context might reflect the overall effectiveness of the educational system or other factors that affect all students equally.

3. Policy implications

The insights from these CES models can inform educational policy in several ways. Starting with the elasticity of substitution (σ), if it is high, policies might focus on developing students' strengths, as deficiencies in one area can be more easily compensated by strengths in another. On the other hand, if σ is low, a more balanced approach to skill development might be necessary, as weaknesses in one area could significantly hinder overall educational outcomes.

The relative magnitudes of α and β can help guide resource allocation. For example, if we find that α is much larger than $(1-\alpha)$, we should prioritize cognitive skill development, always keeping in mind the timing of interventions. Cognition is mostly genetics and proper care during pregnancy and infancy, whereas noncognitive skills can be taught to some extent at any time during the school years.

The returns to scale properties inform us whether policies should focus on "broad-based" improvement of all skills or targeted interventions in specific areas for specific groups (like boys and girls) at specific times, for specific periods.

Research supports my theory that while cognitive skills are more heavily influenced by early childhood experiences and genetics, noncognitive skills remain relatively malleable throughout life. Cognitive skills are significantly impacted by genetics and early childhood experiences. Critical periods for cognitive development occur primarily in early childhood, though some plasticity remains throughout life (Knudsen et al., 2006). Noncognitive skills can be developed and refined throughout the lifespan, including during school years and adulthood (Kautz et al., 2014). This malleability makes noncognitive skills an attractive target for interventions at various life stages.

The early childhood (0-5 years) period is vital for cognitive development, but research suggests that the brain retains some plasticity throughout life. Certain cognitive skills are more easily developed in early childhood (Knudsen et al., 2006). This fact emphasises the importance of early interventions such as the Perry Preschool Project (Schweinhart et al., 2005) and the Abecedarian Project (Campbell et al., 2012), which demonstrated significant improvements in cognitive abilities and later life outcomes through high-quality preschool education.

While early childhood programs remain highly important for cognitive development, some programs targeting noncognitive skills can be effective at various ages, offering opportunities for improvement even later in the educational process. For example, the Chicago School Readiness Project showed improvements in both cognitive and noncognitive skills through preschool interventions (Raver et al., 2011).

The work of James Heckman and colleagues has demonstrated that early childhood interventions can have lasting effects on both cognitive and noncognitive skills, with noncognitive skills often being more malleable later in life (Heckman & Kautz, 2012b). This malleability of noncognitive skills is further supported by research showing that social-emotional or character skills can be developed throughout life, including during school years and even adulthood (Kautz et al., 2014).

It is important to note that while genetics play a role in both cognitive and noncognitive development, the interaction between genes and environment (epigenetics) remains of utmost importance. Proper care during pregnancy and infancy matters for both cognitive and noncognitive development (Fox et al., 2010). Plus, interventions like nurse home visiting programs have been shown to improve cognitive outcomes for children from disadvantaged backgrounds (Olds et al., 2004).

All the aforementioned findings have significant implications for educational policy and practice. They suggest a two-pronged approach: intensive early interventions to support cognitive development, coupled with ongoing programs to foster noncognitive skills throughout the educational journey and beyond. Such a comprehensive strategy may offer the best opportunity to maximize human capital development and improve long-term outcomes for both individuals and society.

J. Empirical considerations

While the CES models provide a nice theoretical framework, their empirical application presented several challenges. The CES function is non-linear in its parameters, which requires non-linear estimation techniques. This has turned out to be computationally intensive and has led to convergence issues in some cases (where the scale starts at zero, for example).

Estimating the elasticity of substitution (σ) or the substitution parameter (ρ) was very challenging, especially because there was limited variation in the ratio of inputs across observations.

Correctly measuring cognitive and noncognitive skills is of extreme importance. Measurement errors can lead to biased estimates of the production function parameters. The fact that the ratings were provided by the Primary caregiver makes it an indirect measure.

As with many empirical models in education, there may be concerns about endogeneity of inputs. For example, higher achieving students might conscientiously (or just by genetic chance) choose to invest more in both cognitive and noncognitive skills.

The CES model imposes specific functional form assumptions that may not always align with the true underlying production process. After all, it is an attempt to approximate concepts through debatable measurement instruments. We are not measuring the mass of the electron, but the realization of internal processes through general tests.