When Do Gender Gaps in Maths Become Skill Gaps? Evidence from Ireland

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Abstract

Research on gender differences in Maths achievement has documented persistent gaps, but most studies examine the gap at a single point in time and say little about whether its composition changes as children develop. This paper asks how the structure of the gender gap in Maths differs between late childhood and early adolescence. Using longitudinal data from Ireland and Oaxaca-Blinder decompositions of the same college-entrance exam outcome with predictors measured in primary school (age 9) and secondary school (age 13), I find that the gap shifts from coefficient-dominant (driven by differences in how similar skills map into outcomes) to endowment-dominant (driven by differences in measured skills themselves) across this transition: in late childhood, boys and girls have similar measured skills but those skills map differently into outcomes; by early adolescence, measured skill differences-principally numeracy-account for most of the gap. An English comparison provides an internal benchmark: the pattern reverses by subject, with girls' advantage remaining coefficient-dominant even when boys hold stronger measured endowments. Distributional decompositions show that average differences mask substantial concentration in the middle of the score distribution. A secondary extension examines father absence under stricter measurement and attrition constraints.

Keywords: Gender gap; Maths achievement; Oaxaca-Blinder decomposition; Father absence; Cognitive skills; Socioeconomic background

JEL Codes: I21, I24, J12, J16

1 Introduction

Gender gaps in Maths achievement are a persistent feature of education systems, though their direction and magnitude vary widely across countries and over time (Fryer & Levitt, 2010; Guiso et al., 2008; Nollenberger et al., 2016). In some settings boys outperform girls; in others the gap has narrowed or reversed. The downstream consequences are also less straightforward than often assumed: a pattern sometimes called the gender-equality paradox holds that countries with greater gender parity tend to have larger, not smaller, gaps in STEM participation, though its measurement and interpretation remain contested-complicating any simple link from school-level Maths differences to labour-market sorting. What is clear is that wherever a gender gap in Maths exists, understanding its structure matters-whether it reflects differences in measured skills or differences in how those skills translate into outcomes has different implications for policy. A large literature has documented the size of these gaps and established that socioeconomic disadvantage makes them worse (Autor et al., 2019; Bertrand & Pan, 2013). But a more basic question has received less attention: does the nature of the gap change as children develop? Is the gap in late childhood, while children are still in primary school, the same kind of gap as the one in early adolescence, after the transition to secondary school-or does something shift in what is driving it?

This paper investigates that question. (More precisely, it asks whether the composition of the gap-the relative contributions of skill differences and differences in how skills map into outcomes-changes between these stages, not whether the gap size itself changes.) Using the Growing Up in Ireland longitudinal study, I decompose the gender gap in the same college-entrance exam outcome using predictors measured at two developmental stages: late childhood (age 9, primary school) and early adolescence (age 13, secondary school). Oaxaca-Blinder decompositions separate the gap into endowments (differences in measured skills) and coefficients (differences in how those skills map into outcomes). What emerges is a compositional shift: in late childhood, the gap is mainly about coefficients-boys and girls look similar on paper, but their characteristics predict different outcomes. By early adolescence, the gap is mainly about endowments-boys have pulled ahead in measured numeracy. A paired bootstrap test confirms the shift ($p < 0.001$). The pattern is consistent with cumulative-advantage models (Cunha & Heckman, 2007), although the decomposition design cannot establish a causal sequence.

Three features give this result context. First, decomposing the same outcome at two ages reveals something that single-age studies cannot see: the gap changes character between childhood and early adolescence. Second, an English comparison provides a subject-specific benchmark-and the pattern reverses entirely, with girls' advantage remaining coefficient-dominant even when boys hold stronger measured endowments. That reversal suggests the timing shift is not a statistical artefact but varies systematically by subject. Third, a secondary extension examines father absence, documenting sizeable associated penalties while keeping interpretation proportional to the stronger measurement and attrition constraints in that section.

Ireland provides a useful setting for this analysis. Its centralised curriculum and standardised national examinations allow consistent achievement measurement across subjects and cohorts. Its gender gaps in adolescent Maths are modest by international standards (Hyde & Mertz, 2009; Lindberg et al., 2010), which makes it a conservative test case: if the composition of even a small gap shifts between developmental stages, the same logic is likely to apply in settings where gaps are larger. Granular linked administrative data on individual Leaving Certificate performance and later educational trajectories are not routinely available to researchers, so the analysis relies on the best feasible survey-based measure in GUI.

Throughout, I treat the coefficients component as descriptive residual variation rather than as a direct measure of discrimination. It can reflect genuine differences in returns, but it can also absorb omitted variables, measurement differences, and functional-form error. The endowments component is more transparent, but it still describes observed characteristics within a selected sample. For that reason, every result in the paper is interpreted as associational.

The main text focuses on decomposition results for the gender-timing pattern in Maths, supported by Figure 1 and distributional analyses. Father-absence results are presented as a secondary descriptive extension under heavier measurement and attrition constraints. Detailed decomposition tables, English comparisons, OLS baselines, and robustness checks (Neumark pooled-coefficient decompositions, design-effect sensitivity, Lee bounds for attrition) are provided in the appendices.

The remainder of the paper is structured as follows. Section II reviews the related literature. Section III describes the data, measurement instruments, and sample construction. Section IV presents the empirical strategy and results. Section V discusses interpretation, limitations, and implications.

2 Related Literature

Three bodies of research motivate this study. The first concerns the structure of gender gaps in Maths. Cross-national evidence establishes that these gaps vary by institutional and cultural context (Fryer & Levitt, 2010; Guiso et al., 2008; Nollenberger et al., 2016), and within countries, boys tend to outperform girls despite showing more behavioural problems and lower school engagement (Bertrand & Pan, 2013). Family disadvantage amplifies these patterns (Autor et al., 2019). Yet most studies examine the gap at a single developmental stage. Fewer ask whether the nature of the gap-whether it reflects differences in skills or differences in how those skills translate into outcomes-changes as children age, even though the answer matters for when and how to intervene.

The second strand concerns skill formation and cumulative advantage. Cunha and

Heckman (2007) formalise two mechanisms: self-productivity, where early skills reinforce later ones, and dynamic complementarity, where early investments raise the returns to subsequent investments. These mechanisms predict that differences in how skills are rewarded at one stage can manifest as differences in skill levels at the next. Evidence on the building blocks is strong: numeracy and verbal reasoning predict later achievement even after accounting for behavioural traits (Duncan et al., 2007), and noncognitive characteristics such as self-regulation matter independently, sometimes rivalling IQ in predictive power (Duckworth & Seligman, 2005). What is less well established is how the relative importance of these inputs changes across developmental stages for subject-specific outcomes like Maths.

The third strand concerns family structure and father absence. Father absence is associated with lower educational attainment and more behavioural difficulties, even conditional on background characteristics (McLanahan et al., 2013). Boys may be more vulnerable to household instability (Fomby & Cherlin, 2007; Lee & McLanahan, 2015), while girls' education may suffer more from reduced parental inputs and monitoring (Brenøe & Lundberg, 2018). School contexts interact with these family dynamics: teacher gender biases influence both performance and subject choices (Lavy & Sand, 2018), and implicit stereotypes can widen gender gaps in Maths and lower girls' confidence (Carlana, 2019). Socioeconomic status further compounds these effects, with children from disadvantaged backgrounds falling further behind as they age (Bradley & Corwyn, 2002; Caro et al., 2009; Sirin, 2005).

This paper sits at the intersection of these three literatures. By decomposing the same outcome at two developmental stages, it addresses the timing question that strand one leaves open, tests whether the observed compositional shift is consistent with the cumulative-advantage mechanisms formalised in strand two, and then examines whether family structure compounds or reshapes these patterns in a secondary extension.

3 Data and Measurement

3.1 The Growing Up in Ireland Longitudinal Study

I use data from the Growing Up in Ireland (GUI) study, the first large nationally representative longitudinal survey of children in Ireland. The Cohort '98 sample comprises 8,568 children born between November 1997 and October 1998, interviewed at ages 9 (Wave 1, 2007–08), 13 (Wave 2, 2011–12), 17/18 (Wave 3, 2015–16), and 20 (Wave 4, 2018–19). Retention remained high: 7,525 at Wave 2, 6,216 at Wave 3, and 5,190 at Wave 4.[1]

The outcome in this study is Leaving Certificate Maths performance at ages 17–18,

[1]At Wave 4, 20-year-olds became the main respondents. This wave collected retrospective information on education, work, and time use, including self-reported Leaving Certificate results.

reported retrospectively at Wave 4. Ideally, the analysis would use linked administrative records on individual Leaving Certificate results and subsequent higher-education trajectories. Access to such data was actively explored but could not be secured for this project, and granular linked administrative data of this kind are not routinely available to researchers in Ireland. The chapter therefore uses the closest feasible measure available in the public GUI data. Predictors are drawn from Wave 1 (age 9) and Wave 2 (age 13), allowing decompositions at two developmental stages predicting the same outcome. Cognitive skills were measured using standardised logit scores from verbal, numerical, and reasoning assessments administered at each wave. Socioemotional traits were assessed using four SDQ difficulty subscales (Emotional Symptoms, Conduct Problems, Hyperactivity/Inattention, Peer Problems), completed by the primary caregiver. Background variables include parental education, equivalised household income, and school characteristics (DEIS status, fee-paying status, co-educational status). Table 1 summarises the timing of data collection and key variables.

Event	Date	Age (in years)	Variables of interest
Study-child is born	Nov/97 - Oct/98	0	-
Wave 1 data collection	Aug/07 - May/08	9	2 Cognitive variables (Reading and Maths logit scores), 4 SDQ scales, Parental Education (mother and father's), Income quintiles, 1 School Indicator (CoEd)
Wave 2 data collection	Aug/11 - Mar/12	13	3 Cognitive variables (Verbal and Numerical logit scores, BAS Matrices), 4 SDQ scales, Parental Education (mother and father's), Income quintiles, 4 School Indicators (DEIS, CoEd, Fee-paying, Religious Ethos)
Study-child sits the Junior Cert	Jun/13 - Jun/15	15–16	-
Wave 3 data collection	Apr/15 - Aug/16	17/18	Most participants had not yet sat the Leaving Cert
Study-child sits the Leaving Cert	Jun/16 - Jun/17	17/18	-
Wave 4 data collection	Aug/18 - Jun/19	20	Leaving Cert points in Maths scores

Table 1: Timeline of events for the Growing Up in Ireland '98 Cohort (birth years 1997/98; baseline sample n = 8,568), showing survey waves, participant ages, and the timing of key outcomes used in the analysis (Junior Certificate and Leaving Certificate Maths points).

The GUI sample was drawn through schools, producing natural clustering at the school level-a design feature addressed in the inference discussion below.[2]

Outcome. Leaving Certificate Maths points are self-reported at Wave 4 (age 20), two to three years after the examination.[3] Because a 2017 reform changed the grading bands, a dummy identifies which system applied to each participant, and bonus points are subtracted where inconsistently reported; the adjusted score, capped at 100, is used

[2]School identifiers are available only through restricted-access Research Microdata Files; school-clustered standard errors are therefore replaced by design-effect sensitivity analysis (Appendix K).

[3]At Wave 3, only 713 participants had already sat the Leaving Cert; most were still in school.

throughout.[4] With a pooled standard deviation of approximately 27 points, the raw gender gap of 4.4–5.2 points corresponds to roughly 0.16–0.19 standard deviations, a small effect by conventional benchmarks (Hyde & Mertz, 2009). A further concern is differential reporting by gender: if boys systematically overreport and girls underreport performance, the observed Maths gap can be mechanically widened. This possibility cannot be ruled out with survey data alone and is therefore treated as a measurement limitation throughout. Prior academic achievement (Junior Certificate grades, mapped to a 12-point scale) is reserved for supplementary analyses because both predictor and outcome are self-reported, creating a risk of correlated measurement error.

Predictors. Socioemotional traits are measured by the four SDQ difficulty subscales (Emotional Symptoms, Conduct Problems, Hyperactivity/Inattention, Peer Problems), completed by the primary caregiver at both waves and by teachers at Wave 1. Parental education enters as dummies for Higher Secondary/Technical and Degree or Postgraduate, separately for each parent; no imputation is performed when father's education is missing. Household income is equivalised using modified OECD weights (1 / 0.66 / 0.33) and entered in quintiles. School-level characteristics-DEIS status, fee-paying status, religious ethos, and co-educational status-are available at Wave 2; only co-educational status is observed at Wave 1.

Table 2 summarises the sample flow from the original GUI cohort to the analytical samples used in each decomposition.

Table 2: Sample flow from original GUI Cohort '98 to analytical samples.

Stage	N	Reason for attrition
Wave 1 baseline (age 9)	8,568	Original cohort
Wave 2 (age 13)	7,525	Non-response
Wave 3 (age 17/18)	6,216	Non-response
Wave 4 (age 20)	5,190	Non-response
Valid LC Maths score	4,333	Item-level missingness
Decomposition samples (complete cases within each specification):		
Gender, W1 no father's educ.	3,690	Covariate missingness
Gender, W1 with father's educ.	3,241	+ father's educ. required
Gender, W2 no father's educ.	3,401	Covariate missingness
Gender, W2 with father's educ.	2,777	+ father's educ. required

[4] Before 2017, bonus points were awarded for Higher Level Maths grades above 40%; the 2017 reform introduced broader H1–H8 / O1–O8 bands with a more uniform points structure.

4 Empirical strategy and results

4.1 Baseline OLS models

I organise the empirical analysis in three steps so that interpretation is cumulative rather than fragmented. I start with baseline OLS models to establish the predictive structure, then move to Oaxaca-Blinder decompositions to separate mean differences into endowments and coefficients, and finally use DFL reweighting to locate where in the outcome distribution those differences are concentrated. Read together, these steps move from association, to composition, to distribution.

To anchor the decomposition results, I first estimate OLS regressions of Leaving Certificate Maths on cognitive, socioemotional, socioeconomic, and school predictors, separately for Wave 1 and Wave 2 (full tables in Appendix B). As expected, predictors measured closer to the outcome are more informative: Wave 2 models explain more variance (adjusted $R^2 \approx 0.39$–0.40) than Wave 1 models ($R^2 \approx 0.30$). Across waves, numerical ability, reading ability, hyperactivity, and parental education remain the most consistent correlates of performance, while missing father's education, used here as a proxy for paternal disengagement, is negatively associated with achievement.

4.2 Design

I estimate separate decompositions using predictors measured at age 9 (Wave 1) and age 13 (Wave 2), while holding the outcome fixed as Leaving Certificate Maths at ages 17–18. This design is intended to show whether the composition of the same gap looks different when observed at different stages of development. Because the predictor sets are not identical across waves, however, I treat wave contrasts carefully: cognitive tests change by wave, and several school controls are only available at Wave 2, so some shift could be mechanical. For that reason, I emphasise component sign, relative magnitude, and the paired bootstrap change test (Table 5), and I also report a harmonised specification restricted to conceptually matched covariates. The harmonised results preserve the core timing pattern, with endowment-share shifts of about 51–62 percentage points in Maths and a reversed shift in English (Appendix N, Table 36). A residual concern is that age-13 cognitive tests may be better-scaled instruments for predicting LC Maths, independently of developmental timing-the Drumcondra Numerical Ability score at age 13 is arguably a more crystallised measure than the Wave 1 Maths logit score. The harmonised specification addresses covariate set composition but cannot fully resolve differences in test quality across waves. The contribution of this design is therefore to document a compositional shift that is robust to covariate harmonisation and that reverses by subject, not to establish a causal developmental sequence.

For each wave, I estimate two models: one including a dummy for missing father's education (using the full sample to capture paternal disengagement), and one controlling explicitly for father's education level (using the subset with complete data). After the gender decompositions, I extend the analysis to father absence, comparing students whose fathers consistently failed to participate in parental surveys at both waves. Separate decompositions by gender capture how boys and girls are differentially affected.

School-clustered standard errors are not available (school identifiers require restricted-access clearance). In school-based surveys, intraclass correlations typically range from 0.10 to 0.25, which can imply design effects substantially above the 1.3–1.5 range tested here. The design-effect exercises therefore show that results are not fragile to modest clustering adjustments, but they do not constitute cluster-robust inference. A partial check is that the English decomposition produces sign reversals relative to Maths: systematic upward bias from unmodelled school clustering would be expected to affect both subjects in the same direction, which is inconsistent with the pattern observed. I report design-effect sensitivity exercises (Appendix K) and note this as a standing limitation. Significance conventions differ between the OLS tables ($^{\dagger}p < 0.1$, $^{*}p < 0.05$, $^{**}p < 0.01$, $^{***}p < 0.001$) and the decomposition tables ($^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$); each table footnote specifies the applicable convention.

4.3 Oaxaca-Blinder decomposition

To decompose the gender gap, I use the threefold Oaxaca-Blinder method (Blinder, 1973; Oaxaca, 1973):

$$\bar{Y}_G - \bar{Y}_B = \underbrace{(\bar{\mathbf{X}}_G - \bar{\mathbf{X}}_B) \cdot \beta_B}_{\text{Endowments}} + \underbrace{\bar{\mathbf{X}}_B \cdot (\beta_G - \beta_B)}_{\text{Coefficients}} + \underbrace{(\bar{\mathbf{X}}_G - \bar{\mathbf{X}}_B) \cdot (\beta_G - \beta_B)}_{\text{Interaction}} \tag{1}$$

The endowments component measures how much of the gap is attributable to differences in measured characteristics between boys and girls, valued at boys' returns. The coefficients component measures how much is attributable to differences in returns to the same characteristics. The interaction captures their joint effect. In all specifications the interaction term is small relative to the endowments and coefficients components, and its sign shifts across waves in the same direction as the endowments component; it is reported in full in the appendix tables and in Figure 1 but is not discussed separately in the text. The decomposition asks a precise question: do boys and girls differ in their skills, or in how those skills translate into achievement?

Two interpretive cautions are essential. First, the coefficients component is a residual. It captures genuine differences in returns alongside omitted variables, measurement error, and functional-form misspecification (Fortin et al., 2011; Jann, 2008). I interpret it descriptively, not as a direct measure of discrimination. Second, the component magnitudes can be sensitive to the reference group (the "index number problem"); I use boys' coefficients as the reference and confirm robustness using the Neumark (1988) pooled-coefficient decomposition (Tables 8 and 9).

4.4 Gender gap decomposition

Figure 1 presents the main result: the gender gap in Maths decomposes very differently depending on the age at which predictors are measured. Detailed variable-level decomposition tables are in Appendix D (Tables 22–23).

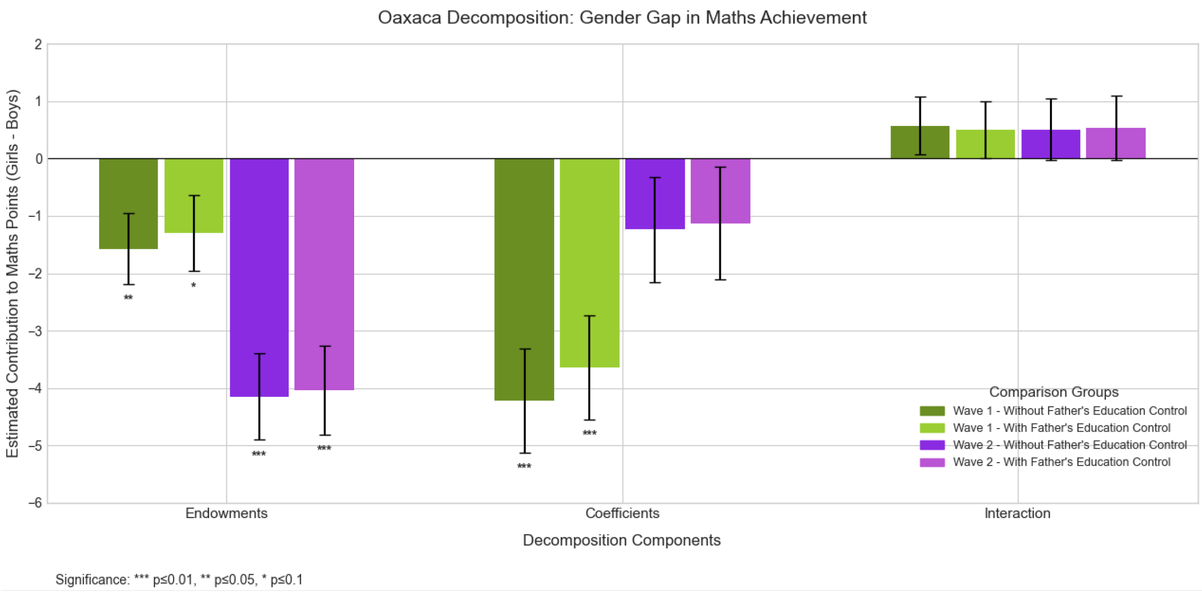

Figure 1: Oaxaca-Blinder decomposition of the gender gap in Leaving Certificate Maths by predictor wave and father-education specification. At age 9, the coefficient component is larger; at age 13, the endowment component is larger. Bars show endowment, coefficient, and interaction terms with bootstrap 95% confidence intervals (1,000 replications). Outcome: adjusted Leaving Certificate Maths points (0–100). Negative values are components associated with boys' higher mean scores. Sample sizes are $n = 3{,}690$, 3,241, 3,401, and 2,777 across panels.

Figure 2 shows the same components as point estimates with confidence intervals, making the wave-to-wave shift easier to read: at age 9 the coefficients component is the largest contributor, while by age 13 the endowments component dominates.

4.5 A comparative test: the gender gap in English

If the coefficients component were simply noise-omitted variables or functional-form misspecification-it should not flip sign systematically across subjects. To test this, I replicate the decomposition for Leaving Certificate English (Appendix F).

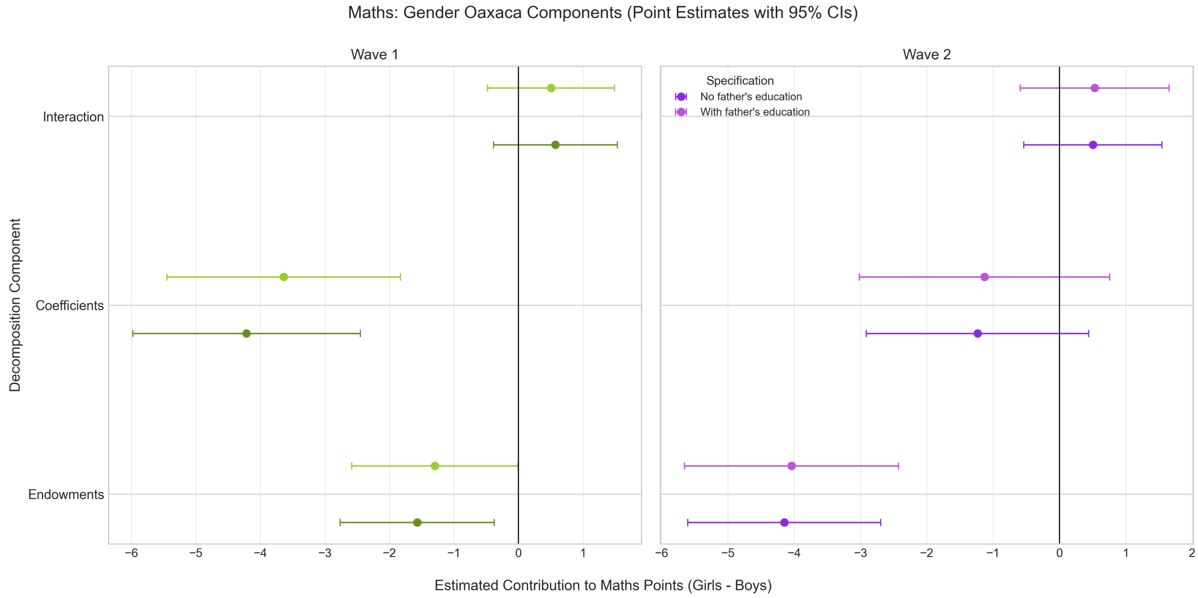

Figure 2: Coefficient-plot representation of the Oaxaca-Blinder decomposition of the gender gap in Leaving Certificate Maths. Points denote component estimates (endowments, coefficients, interaction) and horizontal whiskers denote 95% confidence intervals based on bootstrap standard errors (1,000 replications). Panels separate Wave 1 (age 9 predictors) and Wave 2 (age 13 predictors), and each panel reports estimates with and without father's education controls. Values to the left of zero indicate components associated with higher male scores. Estimates are descriptive and should not be interpreted as causal effects.

The reversal is clear. In English, girls outperform boys by about 3–3.5 points, and the gap is mostly coefficient-dominant. At age 9, girls do not hold a meaningful endowment advantage, so their higher scores are linked mainly to descriptive differences in how similar observed characteristics map into outcomes. By age 13, the pattern becomes even sharper: boys hold a significant endowment advantage (-1.588 points, $p < 0.01$), yet girls continue to score higher because the coefficients component ($+4.548$ points) more than offsets that disadvantage. The Neumark pooled-coefficient decomposition reaches the same conclusion: at Wave 2, endowments account for -40% of the English gap while coefficients account for 140% (Table 9), whereas the comparable Maths decomposition shows endowments at 80% (Table 8).

This cross-subject reversal is consistent with the coefficients component reflecting systematic, context-dependent differences in how observed characteristics map into outcomes. In Maths, the decomposition is endowment-dominant at age 13; in English, it remains coefficient-dominant. The pattern is more consistent with subject-specific decomposition structures than with a blanket "boys are better at X" narrative.

4.6 Father absence

The next question is whether family structure amplifies the patterns seen in the gender results. I treat this as a secondary extension rather than a co-equal pillar, because the father-absence proxy is necessarily noisier and the relevant subsamples are both smaller and more selected than the main gender sample.

Father absence is defined as consistent non-response to the father's questionnaire at both Wave 1 and Wave 2, capturing sustained paternal disengagement throughout childhood and early adolescence. This proxy combines structural absence and low engagement among resident partners. Wave-3 validation shows that the father-absent group is predominantly structurally absent (61.2% with no resident partner), with smaller shares of resident-but-disengaged (12.3%) and engaged-resident (6.0%) cases (Appendix C, Table 21). Alternative definitions separating structural absence from resident disengagement preserve the baseline conclusion (Appendix N, Table 38).

Father absence is strongly patterned rather than randomly distributed. Relative to father-present students, father-absent students have lower parental education, lower household income, weaker cognitive scores, and more socioemotional difficulties (Table 14). They are also much more likely to attrit before Wave 4 (36% valid LC scores vs. 64% for father-present; $\chi^2 = 320.52$, $p < 0.001$), which implies that observed penalties may understate the full disadvantage. The decomposition then asks how much of the remaining gap is associated with measured characteristics and how much with differences in returns.

Figure 3 presents the results by gender and wave (variable-level detail in Appendix E).

4.7 Distributional decomposition

The Oaxaca-Blinder decompositions presented above focus on mean differences. To examine how gaps vary across the achievement distribution, I implement DiNardo-Fortin-Lemieux (DFL) reweighting decompositions (DiNardo et al., 1996) at the 10th, 25th, 50th, 75th, and 90th percentiles, with bootstrap confidence intervals from 1,000 stratified replications. I adopt DFL over the recentered influence function (RIF) method of Firpo et al. (2009) because Leaving Certificate Maths points exhibit mass points from the discrete grade-band structure, making the kernel density estimation required by RIF unreliable; details are in Appendix H.

Gender gap in Maths. Figure 4 and Table 3 show that the mean gap of 4–5 points hides substantial internal structure. Around the middle of the distribution, boys outperform girls by 10–21 points depending on specification, whereas at the 10th and 90th percentiles the gap is small (0–5 points) and often imprecise. With age-9 predictors, the composition effect is close to zero across most quantiles; with age-13 predictors, it becomes materially

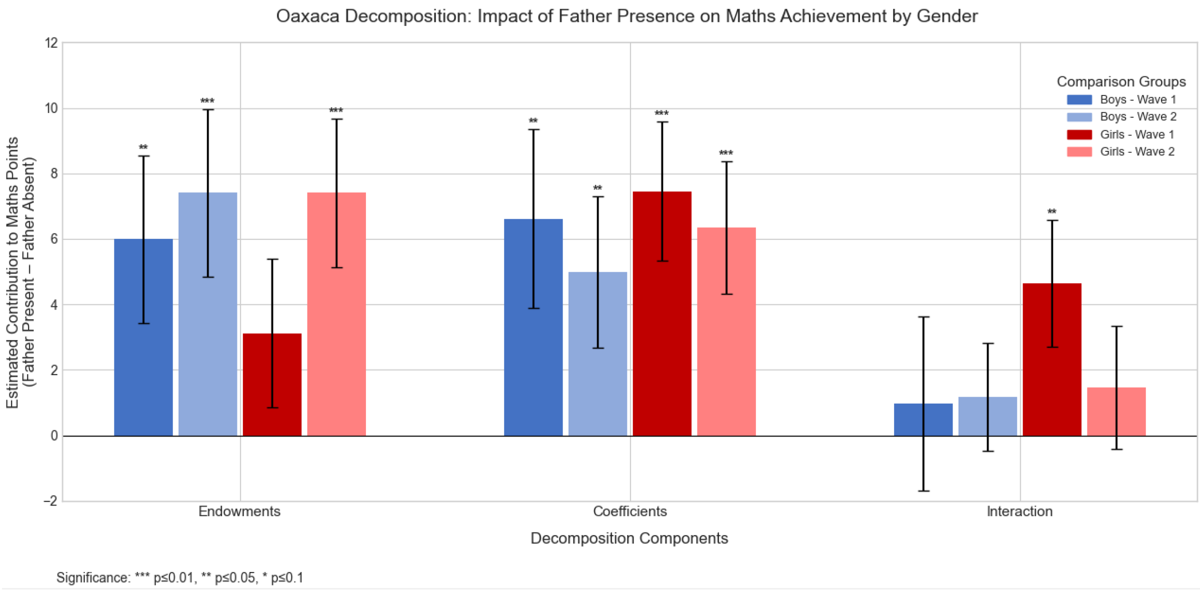

Figure 3: Oaxaca-Blinder decomposition of father-absence differences in Leaving Certificate Maths, estimated separately by gender and predictor wave. Positive values indicate higher mean scores among father-present students. Bars show endowment, coefficient, and interaction terms with bootstrap 95% confidence intervals (1,000 replications). Group means are 60.83 vs 47.27 for boys ($n = 1{,}314$) and 55.80 vs 40.57 for girls ($n = 1{,}292$).

larger, accounting for 100% of the gap at the 25th percentile and 29% at the median. This is consistent with the mean-decomposition result that endowments matter more by age 13. At the same time, the distributional evidence must be interpreted alongside level selection: support-split and extensive-intensive results indicate that part of the median contrast reflects differential placement between Higher and Ordinary level Maths, not only within-level performance. Even under that stricter interpretation, the Wave 1 to Wave 2 compositional shift remains visible (Appendix L; Appendix N, Table 37). The extensive-intensive decomposition (Appendix N) separates whether the level-mix effect operates through the probability of sitting Higher Level Maths (extensive margin) or through points conditional on level (intensive margin). The endowment-share shift is present on both margins, indicating that the timing result is not reducible to gender differences in level selection alone.

Gender gap in English. The DFL decomposition for English (Table 4) mirrors the Oaxaca-Blinder reversal. The mean gap (3.1 points favouring girls) is concentrated at the 10th percentile (10 points with age-9 predictors) and is negligible from the median upward. With age-13 predictors, the composition effect turns negative at the mean (-1.49 points, SE $= 0.50$): boys have better measured characteristics on average, yet girls still score higher. The coefficient component accounts for 151% of the mean gap at age 13, consistent with the subject-specific interpretation from the Oaxaca-Blinder analysis. Detailed figures and tables are in Appendix I.

Table 3: DFL (DiNardo-Fortin-Lemieux) quantile decomposition of the gender gap in adjusted LC Maths points. The gap is largest at the median and smaller at the tails. With age-13 predictors (Wave 2), composition effects account for a larger share of the median gap than with age-9 predictors (Wave 1). Bootstrap SEs from 1,000 stratified replications.

Specification	Component	$q_{0.10}$	$q_{0.25}$	$q_{0.50}$	$q_{0.75}$	$q_{0.90}$
W1 no father ed.	Total gap	0.0	5.0	10.0	7.0	4.0
		(2.5)	(2.6)	(4.2)	(1.3)	(0.3)
	Composition	0.0	0.0	0.0	0.0	0.0
		(0.4)	(0.8)	(0.5)	(1.9)	(0.3)
	Composition %	—	0%	0%	0%	0%
W1 with father ed.	Total gap	5.0	5.0	14.0	7.0	4.0
		(2.7)	(2.5)	(5.5)	(2.1)	(0.4)
	Composition	0.0	0.0	0.0	4.0	0.0
		(2.4)	(1.8)	(2.0)	(2.0)	(0.6)
	Composition %	0%	0%	0%	57%	0%
W2 no father ed.	Total gap	5.0	5.0	14.0	7.0	4.0
		(1.8)	(2.0)	(5.2)	(1.4)	(0.4)
	Composition	5.0	5.0	4.0	4.0	0.0
		(2.5)	(1.6)	(3.0)	(1.5)	(1.2)
	Composition %	100%	100%	29%	57%	0%
W2 with father ed.	Total gap	0.0	2.0	21.0	3.0	4.0
		(2.2)	(2.3)	(5.4)	(1.7)	(1.0)
	Composition	0.0	2.0	6.0	0.0	0.0
		(2.1)	(2.2)	(4.5)	(1.7)	(1.8)
	Composition %	—	100%	29%	0%	0%

Notes: Standard errors in parentheses are from 1,000 stratified bootstrap replications. At each quantile, *Total gap* is boys minus girls. *Composition* is the counterfactual-minus-girls component (differences in observed characteristics), and *Structure* is the residual component such that *Total* = *Composition* + *Structure*. *Composition %* is defined as Composition/Total and is omitted when the total gap is zero (or very close to zero). "No father ed." includes a missing-father-education indicator; "with father ed." restricts to observations with observed father's education. Outcome: adjusted LC Maths points (0–100). Sample sizes vary by specification (see decomposition tables/sample-flow table).

Table 4: DFL (DiNardo-Fortin-Lemieux) quantile decomposition of the gender gap in adjusted LC English points. The gap favours girls and is concentrated at the lower tail; with age-13 predictors (Wave 2), composition effects are negative at several quantiles, indicating stronger measured endowments for boys even where girls retain a score advantage. Bootstrap SEs from 1, 000 stratified replications.

Specification	Component	$q_{0.10}$	$q_{0.25}$	$q_{0.50}$	$q_{0.75}$	$q_{0.90}$
Wave 1 (age 9 predictors)						
W1 no father ed.	Total gap	10.0	4.0	0.0	0.0	0.8
		(3.6)	(1.0)	(1.7)	(1.1)	(1.0)
	Composition	2.0	0.0	0.0	0.0	0.0
		(2.6)	(0.5)	(1.1)	(0.7)	(0.1)
	Composition %	20%	0%	—	—	0%
W1 with father ed.	Total gap	9.0	4.0	5.0	0.0	2.0
		(2.2)	(1.4)	(2.7)	(2.4)	(0.9)
	Composition	0.0	0.0	0.0	0.0	0.0
		(1.9)	(0.8)	(1.0)	(0.5)	(0.2)
	Composition %	0%	0%	0%	—	0%
Wave 2 (age 13 predictors)						
W2 no father ed.	Total gap	8.0	4.0	0.0	0.0	2.0
		(2.0)	(1.7)	(2.5)	(2.3)	(1.0)
	Composition	-2.5	0.0	-4.0	-3.0	0.0
		(2.8)	(1.2)	(2.0)	(1.4)	(0.3)
	Composition %	-31%	0%	—	—	0%
W2 with father ed.	Total gap	6.0	9.0	5.0	5.0	2.0
		(1.9)	(2.8)	(1.0)	(2.0)	(0.9)
	Composition	-3.0	0.0	0.0	0.0	0.0
		(1.8)	(1.8)	(1.7)	(1.4)	(0.4)
	Composition %	-50%	0%	0%	0%	0%

Notes: Standard errors in parentheses are from 1,000 stratified bootstrap replications. At each quantile, *Total gap* is girls minus boys (positive values indicate a girls' advantage). *Composition* is the counterfactual-minus-boys component (differences in observed characteristics), and *Structure* is the residual component such that *Total* = *Composition* + *Structure*. *Composition %* is defined as Composition/Total and is omitted when the total gap is zero (or very close to zero). Negative composition values indicate endowments that would favour boys absent offsetting structure effects. "No father ed." includes a missing-father-education indicator; "with father ed." restricts to observations with observed father's education. Outcome: adjusted LC English points. Sample sizes vary by specification (see sample-flow and decomposition tables).

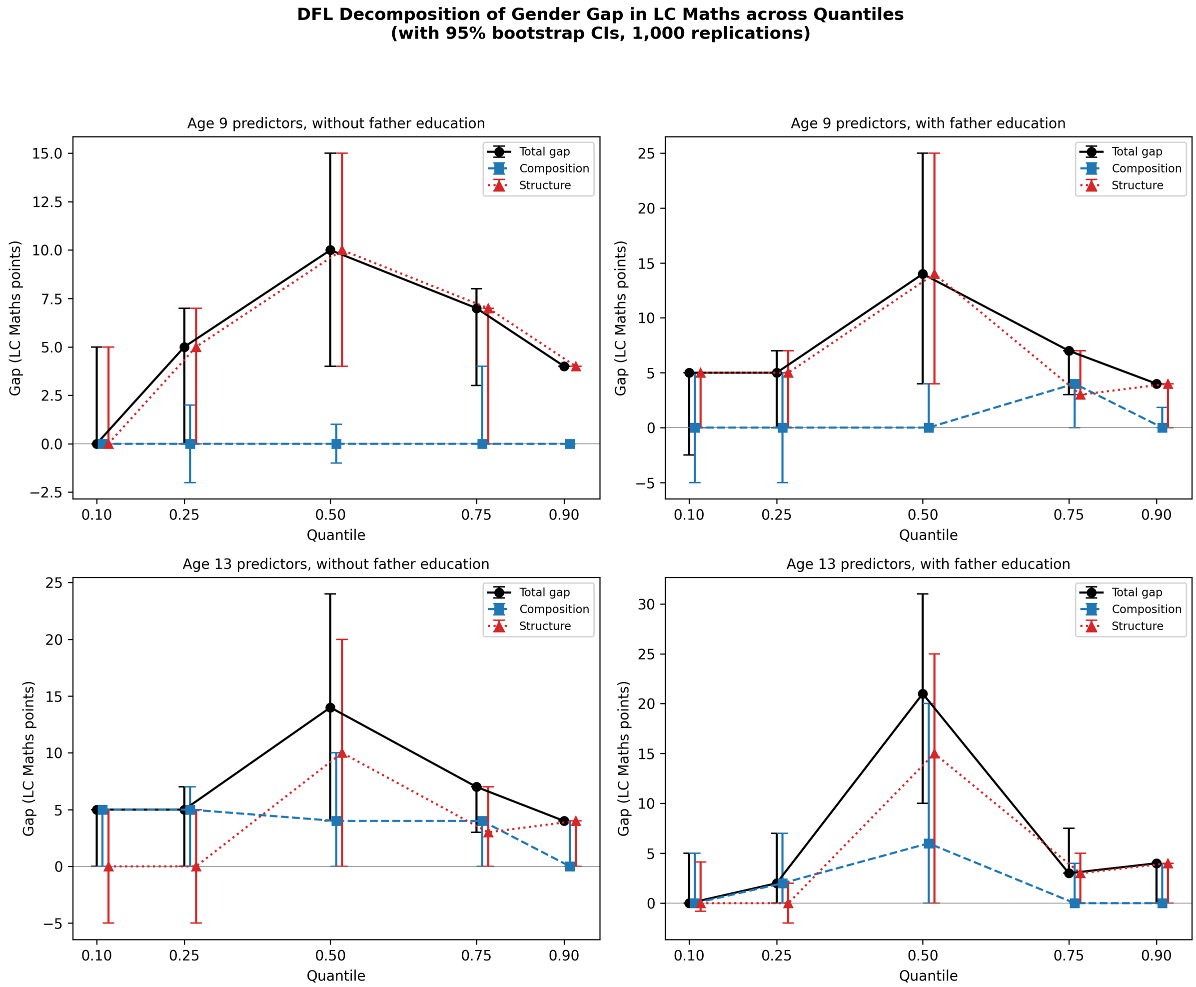

Figure 4: DFL decomposition of the gender gap in LC Maths across quantiles, with 95% bootstrap confidence intervals (1,000 stratified replications). Top row: age 9 predictors; bottom row: age 13 predictors; left column excludes father's education; right column includes it. Black circles: total gap (boys − girls). Blue squares: composition effect (counterfactual − girls). Red triangles: structure effect (boys − counterfactual). The gender gap is concentrated at the median across all specifications, with composition effects becoming more important when age 13 predictors are used. Sample: $N = 4{,}333$ (complete cases vary by specification). Outcome: adjusted LC Maths points (0–100 scale).

Father-absence gaps. For father absence in Maths, the penalty is broadly distributed across quantiles rather than concentrated at the median: boys show the largest gap at the median (−28.5 points, SE = 6.3) with the composition effect explaining 88–91%, while for girls the gap peaks at the 75th percentile (−29 points) with the composition effect varying sharply by wave (11% with age-9 predictors, 61% with age-13 predictors). In English, the father-absence penalty is smaller but shows a distinctive feature for girls: with age-13 predictors, the composition effect (−5.52 points) exceeds the total gap (−4.18 points), implying that measured-skill differences between father-present and father-absent girls more than account for the overall penalty. Bootstrap tables for both subjects are reported in Appendix H (Maths) and Appendix J (English).

4.8 Robustness

I assess the timing claim with a paired bootstrap that resamples the same individuals across waves. This matters because it anchors the comparison in within-sample changes rather than across-sample composition. Table 5 shows that, in the primary Maths specification, the endowment component rises by +2.51 points from Wave 1 to Wave 2 (95% CI: $[1.52, 3.50]$, $p < 0.001$), and the endowment share increases by 43 percentage points, from 32% at age 9 to 75% at age 13. In English, the sign flips in the opposite direction: the endowment component falls by -1.58 points ($p < 0.001$). For father-absence decompositions in Maths, the shifts are directionally similar but statistically imprecise ($p = 0.21$–0.34). The father-absence extension therefore does not carry a timing claim: its contribution is to document a sizeable mean penalty (14–15 points) with reasonable attrition robustness, not to establish a compositional shift across developmental stages.

Table 5: Paired bootstrap timing test of Wave 1 vs Wave 2 decomposition differences. Gender decompositions show a statistically significant shift from coefficients to endowments ($p < 0.001$), while father-absence shifts are not statistically significant. Endowment Δ is the change in the endowment component from Wave 1 to Wave 2; the coefficients component is the exact complement ($-\Delta$) by construction.

Specification	Endow. Δ	95% CI	p	Share Δ (pp)	N
Maths, no father ed.	$+2.51^{***}$	[1.52, 3.50]	0.000	$+43.1^{***}$	2,964
Maths, with father ed.	$+2.34^{***}$	[1.15, 3.52]	0.000	$+46.5^{***}$	2,330
English, no father ed.	-1.58^{***}	[−2.19, −0.96]	0.000	-53.8^{***}	2,944
English, with father ed.	-1.34^{***}	[−2.04, −0.66]	0.000	-36.7^{***}	2,316
FA Maths, Boys	+1.67	[−0.82, 4.08]	0.208	+12.3	1,314
FA Maths, Girls	+1.21	[−1.35, 3.42]	0.336	+7.9	1,292
FA English, Boys	+0.22	[−1.28, 1.84]	0.798	+3.9	1,300
FA English, Girls	$+1.28^{*}$	[−0.13, 2.56]	0.072	$+30.7^{*}$	1,289

Notes: Paired bootstrap test of H_0: $\text{component}_{\text{W2}} = \text{component}_{\text{W1}}$, using Neumark pooled-coefficient decompositions. The same individuals are resampled in each replication, preserving within-person dependence across waves. 1,000 bootstrap replications; 95% percentile CIs. Δ = W2 minus W1 estimate. Share Δ reports the change in endowment share of the total gap (percentage points). The coefficients Δ is the exact complement ($-\Delta$) in every row. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.10$.

Harmonised covariate specification. A natural concern is that the Wave 1 to Wave 2 shift reflects changing predictor sets rather than developmental timing. Cognitive tests differ across waves (BAS-derived measures at age 9; Drumcondra Numerical Ability and Verbal Reasoning at age 13), and several school controls are only available at Wave 2. To address this, I re-estimate the decomposition using only conceptually matched covariates that exist at both waves: a numeracy measure, a reading/verbal measure, the four SDQ subscales, primary caregiver education, equivalised household income, school type, and a father-education-missing indicator. With these harmonised covariate sets,

Table 6: Quantile-level timing test (DFL, approximate z-test): differences between Wave 2 and Wave 1 in composition and structure components. Positive composition Δ means the composition effect explains more of the gap at age 13 than at age 9.

	Composition Δ (W2−W1)			Structure Δ (W2−W1)		
Quantile	Est.	SE	p	Est.	SE	p
Gender, Maths (no father ed.)						
Mean	$+2.10^{*}$	1.08	0.052	-2.43^{*}	1.41	0.084
0.10	$+5.00^{**}$	2.50	0.045	$+0.00$	3.65	1.000
0.25	$+5.00^{***}$	1.79	0.005	-5.00	3.21	0.120
0.50	$+4.00$	3.04	0.188	$+0.00$	6.78	1.000
0.75	$+4.00$	2.44	0.101	-4.00	2.46	0.104
0.90	$+0.00$	1.27	1.000	$+0.00$	1.35	1.000
Gender, Maths (with father ed.)						
Mean	$+1.91$	1.17	0.103	-1.71	1.51	0.258
0.10	$+0.00$	3.22	1.000	-5.00^{*}	2.87	0.082
0.25	$+2.00$	2.85	0.482	-5.00^{*}	2.61	0.055
0.50	$+6.00$	4.91	0.221	$+1.00$	8.39	0.905
0.75	-4.00	2.61	0.125	$+0.00$	1.84	1.000
0.90	$+0.00$	1.95	1.000	$+0.00$	2.11	1.000
Gender, English (no father ed.)						
Mean	-1.98^{***}	0.65	0.002	$+1.80^{*}$	0.92	0.051
0.10	-4.49	3.87	0.246	$+2.49$	4.92	0.613
0.25	$+0.00$	1.34	1.000	$+0.00$	2.18	1.000
0.50	-4.00^{*}	2.26	0.077	$+4.00$	3.35	0.233
0.75	-3.00^{*}	1.60	0.062	$+3.00$	2.87	0.295
0.90	$+0.00$	0.34	1.000	$+1.20$	1.40	0.391
Gender, English (with father ed.)						
Mean	-1.68^{**}	0.66	0.011	$+1.25$	0.98	0.205
0.10	-3.00	2.66	0.259	$+0.00$	3.04	1.000
0.25	$+0.00$	2.00	1.000	$+5.00^{*}$	2.75	0.069
0.50	$+0.00$	2.01	1.000	$+0.00$	3.14	1.000
0.75	$+0.00$	1.48	1.000	$+5.00$	3.29	0.128
0.90	$+0.00$	0.41	1.000	$+0.00$	1.21	1.000

Notes: Approximate z-tests use $SE_\Delta = \sqrt{SE^2_{W1} + SE^2_{W2}}$ (conservative under positive correlation). Δ = W2 estimate minus W1 estimate. 1,000 bootstrap replications per wave. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.10$.

the endowment-share shift remains large and statistically significant: approximately 51 percentage points in Maths and 54 percentage points (reversed) in English (all $p < 0.001$; Table 36 in Appendix N). The timing pattern is therefore not an artefact of richer Wave 2 measurement; it survives when the predictor structure is held constant across waves.

Reference-group sensitivity. The Neumark (1988) pooled-coefficient decomposition confirms that results are robust to the reference-group choice across all specifications (Tables 8–11). Table 7 consolidates the evidence from all robustness checks for the main gender-timing result.

Table 7: Master robustness summary for the main gender-timing result in LC Maths.

Check	Evidence for the Wave 1 → Wave 2 shift
Standard Oaxaca-Blinder (main specification)	Wave 1 endowment share: 32%; Wave 2 endowment share: 75%; paired-bootstrap shift: +43 percentage points ($p < 0.001$).
Neumark pooled-coefficient decomposition	Wave 1 endowment share: 28%; Wave 2 endowment share: 80%; confirms reference-group invariance of the timing pattern.
Harmonised wave covariates	Restricting both waves to matched blocks (cognition, SDQ, SES, school-mix, grading dummy) still gives a large shift: +51 to +62 percentage points (all $p < 0.001$).
Level-mix decomposition (extensive vs intensive)	At Wave 2, endowments dominate the extensive margin (upper-support placement: ∼89% share), while within-upper-support points gaps are smaller and less endowment-dominant (∼18–20% share).
School-clustering inference sensitivity	Under design-effect inflation (×1.3, ×1.5), the headline component ordering is unchanged: Wave 1 remains coefficient-led and Wave 2 remains endowment-led.

Note: Entries summarise Tables 5, 8, 36, 37, and 32. All figures are descriptive decomposition results.

Selection and attrition. IPW reweighting finds no evidence of selection bias on observables: all 12 specifications preserve signs and magnitudes (Table 12). Lee bounds for the father-absence Maths gap remain negative in all samples ($[-30.55, -5.73]$ overall; Appendix M). For the main gender decomposition, Lee bounding is less applicable because attrition rates do not differ significantly by gender; however, the overall sample loss from the baseline cohort (57–68% depending on specification) means results represent children who remained in the study through age 20, who are systematically more advantaged. All findings are interpreted conditional on this selected sample.

Table 8: Endowments explain 28–30% of the Maths gender gap at age 9 but 80–84% at age 13. Neumark (1988) pooled-coefficient decomposition of adjusted LC Maths points using the gender-combined OLS coefficient vector as reference, eliminating the index-number problem.

	Gap	Endowments (pooled $\hat{\beta}$)	Coefficients (total unexplained)	Endow. %
Maths, W1 no father ed.	5.212***	1.469***	3.744***	28%
	(0.922)	(0.534)	(0.873)	
Maths, W1 with father ed.	4.434***	1.318**	3.116***	30%
	(1.029)	(0.625)	(0.806)	
Maths, W2 no father ed.	4.882***	3.907***	0.975	80%
	(1.022)	(0.676)	(0.777)	
Maths, W2 with father ed.	4.635***	3.876***	0.759	84%
	(1.204)	(0.725)	(0.845)	

Notes: Neumark (1988) pooled-coefficient decomposition. The pooled (gender-combined or treatment-combined) OLS coefficient vector is used as the reference. Endowments = $(\bar{X}_A - \bar{X}_B)\hat{\beta}_{\text{pool}}$. Coefficients = advantage to group A plus disadvantage to group B. Bootstrap standard errors in parentheses (100 replications). $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table 9: In English, endowments favour boys (−40% at Wave 2), yet girls score higher because their coefficient advantage (+140%) more than compensates. Neumark (1988) pooled-coefficient decomposition of adjusted LC English points.

	Gap	Endowments (pooled $\hat{\beta}$)	Coefficients (total unexplained)	Endow. %
English, W1 no father ed.	3.079***	0.666*	2.413***	22%
	(0.636)	(0.399)	(0.518)	
English, W1 with father ed.	3.566***	0.776**	2.790***	22%
	(0.607)	(0.345)	(0.552)	
English, W2 no father ed.	2.906***	-1.169***	4.075***	-40%
	(0.678)	(0.404)	(0.503)	
English, W2 with father ed.	3.135***	-0.815*	3.950***	-26%
	(0.733)	(0.434)	(0.613)	

Notes: Neumark (1988) pooled-coefficient decomposition. The pooled (gender-combined or treatment-combined) OLS coefficient vector is used as the reference. Endowments = $(\bar{X}_A - \bar{X}_B)\hat{\beta}_{\text{pool}}$. Coefficients = advantage to group A plus disadvantage to group B. Bootstrap standard errors in parentheses (100 replications). $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table 10: Endowments account for 44–55% of the father-absence Maths gap at age 9 and 55–58% at age 13. Neumark (1988) pooled-coefficient decomposition of adjusted LC Maths points, estimated separately by gender.

	Gap	Endowments (pooled $\hat{\beta}$)	Coefficients (total unexplained)	Endow. %
Maths, W1, Boys	13.564***	7.127***	6.438***	53%
	(2.847)	(1.762)	(2.429)	
Maths, W1, Girls	15.225***	7.672***	7.553***	50%
	(2.687)	(1.720)	(2.123)	
Maths, W1, All	14.736***	7.709***	7.026***	52%
	(2.073)	(1.281)	(1.592)	
Maths, W2, Boys	13.564***	8.793***	4.771**	65%
	(2.849)	(1.848)	(2.245)	
Maths, W2, Girls	15.225***	8.881***	6.344***	58%
	(2.440)	(1.917)	(1.701)	
Maths, W2, All	14.736***	9.215***	5.520***	63%
	(1.829)	(1.326)	(1.490)	

Notes: Neumark (1988) pooled-coefficient decomposition. The pooled (gender-combined or treatment-combined) OLS coefficient vector is used as the reference. Endowments = $(\bar{X}_A - \bar{X}_B)\hat{\beta}_{\text{pool}}$. Coefficients = advantage to group A plus disadvantage to group B. Bootstrap standard errors in parentheses (100 replications). $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table 11: The father-absence penalty in English is smaller than in Maths and is predominantly endowment-driven. Neumark (1988) pooled-coefficient decomposition of adjusted LC English points, estimated separately by gender.

	Gap	Endowments (pooled $\hat{\beta}$)	Coefficients (total unexplained)	Endow. %
English, W1, Boys	5.673***	3.760***	1.913	66%
	(2.025)	(1.019)	(1.707)	
English, W1, Girls	4.179**	3.408***	0.770	82%
	(1.660)	(0.968)	(1.446)	
English, W1, All	4.621***	3.256***	1.365	70%
	(1.352)	(0.766)	(1.011)	
English, W2, Boys	5.673***	3.979***	1.694	70%
	(2.058)	(1.087)	(1.568)	
English, W2, Girls	4.179**	4.692***	-0.514	112%
	(1.774)	(0.915)	(1.391)	
English, W2, All	4.621***	4.035***	0.586	87%
	(1.326)	(0.581)	(1.070)	

Notes: Neumark (1988) pooled-coefficient decomposition. The pooled (gender-combined or treatment-combined) OLS coefficient vector is used as the reference. Endowments = $(\bar{X}_A - \bar{X}_B)\hat{\beta}_{\text{pool}}$. Coefficients = advantage to group A plus disadvantage to group B. Bootstrap standard errors in parentheses (100 replications). $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Table 12: All 12 decomposition specifications are insensitive to inverse-probability weighting: signs, magnitudes, and significance levels are unchanged. IPW sensitivity analysis comparing unweighted and inverse-probability-weighted Oaxaca-Blinder decompositions across gender and father-absence specifications.

	Endowments		Coefficients		Change (%)	
Specification	UW	IPW	UW	IPW	Endow.	Coef.
G Maths, W1	1.04*	1.04	3.71***	3.71***	0.0	0.0
G Maths, W2	3.60***	3.60***	0.95	0.95	0.0	0.0
G English, W1	0.91**	0.91**	2.97***	2.97***	0.0	0.0
G English, W2	-1.45***	-1.45***	5.00***	5.00***	0.0	0.0
FA Maths, W1, Boys	6.95***	6.95***	6.61***	6.61**	0.0	0.0
FA Maths, W1, Girls	7.77***	7.77***	7.46***	7.46***	0.0	0.0
FA Maths, W2, Boys	8.57***	8.57***	4.99*	4.99*	0.0	0.0
FA Maths, W2, Girls	8.87***	8.87***	6.35***	6.35***	0.0	0.0
FA English, W1, Boys	3.74***	3.74***	1.93	1.93	0.0	0.0
FA English, W1, Girls	3.44***	3.44***	0.74	0.74	0.0	0.0
FA English, W2, Boys	3.70***	3.70***	1.97	1.97	0.0	0.0
FA English, W2, Girls	4.86***	4.86***	-0.68	-0.68	0.0	0.0
Sign stability	Endowments: 12/12; Coefficients: 12/12					
Median $\lvert\Delta\rvert$	Endowments: 0.0%; Coefficients: 0.0%					

Notes: UW = unweighted OB decomposition; IPW = inverse-probability-weighted. Selection model: logit predicting observability (valid LC score) from Wave 1 covariates. Weights trimmed at 1st/99th percentiles and normalised. Bootstrap SEs from 100 replications. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

5 Discussion and conclusion

The main finding is straightforward: the gender gap in Maths changes character between childhood and early adolescence. At age 9, boys and girls look similar on measured skills, but the same characteristics predict different outcomes. By age 13, the picture has shifted-boys have pulled ahead on measured numeracy, and that skill difference now accounts for most of the gap. The gap does not just persist; it becomes a different kind of gap. Three findings matter most for how this pattern should be interpreted.

First, the compositional shift. The decompositions show a clear transition from a coefficient-dominant gap at age 9 to an endowment-dominant gap at age 13, and this shift survives harmonised covariate specifications, alternative decomposition methods, and a paired bootstrap test. The English comparison reinforces the finding: in English, the pattern reverses entirely, with girls' advantage remaining coefficient-dominant even when boys hold stronger measured endowments. That subject-specific reversal is consistent with the decomposition capturing systematic differences in how observed characteristics map into outcomes, not statistical noise.

Second (secondary extension), father absence is descriptively associated with large penalties, but through different decomposition channels depending on gender. The 14–15 point achievement gap associated with father absence operates through both endowments and coefficients, but the composition differs: for boys, both lower measured skills and the coefficients component contribute; for girls, the coefficients component-particularly the terms associated with maternal education and household income-accounts for more of the gap (Brenøe & Lundberg, 2018). Given proxy construction and attrition constraints, this extension is interpreted as supportive descriptive evidence rather than a co-equal core result.

Third, the mean gap masks distributional concentration at the median. The gender gap reaches 10–21 points around the median while remaining comparatively modest at both tails. This means that average-based summaries can misstate where constraints are most acute, and it suggests that interventions focused only on the extremes would miss the centre of the distribution where the largest differences emerge.

5.1 Mechanisms

Several overlapping channels are consistent with-but not proven by-the observed patterns. Teacher expectations and implicit biases may differentially reward boys' and girls' skills early in schooling (Carlana, 2019; Lavy & Sand, 2018). Stereotype threat can lower girls' Maths confidence and engagement during adolescence (Cimpian et al., 2016; Spencer et al., 1999). Self-efficacy gaps-girls often report lower Maths confidence even at similar performance levels (Whitcomb et al., 2020)-can affect subject choices, persistence, and effort. Crucially, these channels are not independent. Early differences in how observed characteristics map into outcomes may shape peer environments and self-beliefs, initiating a cumulative process (Cunha & Heckman, 2007) in which a coefficient-dominant gap at age 9 consolidates into an endowment-dominant gap by age 13. The compositional shift documented here is consistent with this trajectory, though the decomposition design cannot establish the causal sequence directly.

5.2 Limitations

Three categories of limitation shape interpretation.

Identification and inference. The design is associational; all findings should be interpreted as descriptive decompositions, not causal effects. The coefficients component is a residual that captures omitted variables, measurement differences, and model misspecification alongside genuine differences in returns. School-clustered standard errors are not available (school identifiers require restricted access); design-effect sensitivity exercises confirm that the main conclusions survive plausible clustering adjustments (Appendix K). GUI wave-specific survey weights exist, but consistent weights across all four decomposition samples (which span Waves 1, 2, and 4 with varying missingness patterns) are not available in the public-use file; results are therefore unweighted conditional associations. IPW reweighting on observables (Table 12) provides a partial check on selection, but residual selection on unobservables cannot be excluded.

Measurement. The outcome is self-reported Leaving Certificate Maths scores collected two to three years after the exam, so recall error and social-desirability bias remain possible. A specific concern is asymmetric reporting by gender, since over-reporting by boys or under-reporting by girls would mechanically widen measured gaps. The score also amalgamates Higher and Ordinary Level results, and gendered level selection may inflate distributional contrasts at the median (Appendix L). Finally, the father-absence proxy (non-response at both waves) mixes structural absence, resident disengagement, and missing data (Appendix C, Table 21). This limitation reflects the Irish data environment rather than a preference for survey-based outcomes over linked administrative examination records. A reporting-diagnostic analysis (Appendix P) confirms that the LC Maths distribution reflects grade-boundary discretisation rather than free self-report, which limits the scope for idiosyncratic misreporting.

Selection and attrition. Retained children score 0.3–0.4 standard deviations higher on Wave 1 cognitive tests and come from more educated, higher-income households than those lost to attrition; the pattern is similar for boys and girls (Appendix O). Father-absent students are significantly more likely to attrit (36% valid outcomes vs. 64% for father-present), so the analytical sample over-represents more advantaged father-absent students. IPW reweighting and Lee bounds both indicate that the father-absence penalty survives under selection on observables and monotonic attrition assumptions, but residual selection on unobservables cannot be ruled out.

Despite these limitations, the key patterns-coefficient-dominant gap at age 9, endowment-dominant gap at age 13, subject-specific reversal in English, gendered channels of father-absence effects-replicate across decomposition methods (standard O-B, Neumark, DFL), robustness checks (IPW, Lee bounds, harmonised wave comparisons, design-effect inflation), and outcome definitions.

5.3 Implications

These findings have economic significance beyond the classroom. Maths achievement is a strong predictor of STEM entry, and the early gaps documented here may contribute to women's under-representation in fields where wage returns are high (Card & Payne, 2021). The secondary father-absence results are consistent with additional disadvantage among vulnerable households, but should be interpreted with greater caution given measurement and selection constraints (Chetty et al., 2020). At the aggregate level, Hsieh et al. (2019) estimate that better talent allocation-including reducing barriers for women in STEM-accounted for 20–40% of U.S. economic growth from 1960 to 2010.

The timing result suggests directions for intervention design. If the gap is primarily coefficient-dominant at age 9, then programmes that address how boys' and girls' observed characteristics translate into outcomes-for example, reducing teacher bias (Lavy & Sand, 2018) or building noncognitive skills like patience and risk-taking (Alan & Ertac, 2018)-may warrant investigation during middle childhood, before the decomposition becomes endowment-dominant. The distributional concentration at the median further suggests that middle-performing girls may be a policy-relevant group to study. These are hypotheses for causal evaluation, not conclusions the current design can establish.

In summary, the gender gap in Maths is not static-it changes character between childhood and early adolescence. At age 9, the gap is not mainly about who has more skill; by age 13, it is. That shift, together with the English reversal, suggests that timing matters for how gender inequality in achievement is structured and not only for how large it is. These patterns are consistent with the hypothesis that gender differences in how measured characteristics translate into outcomes precede differences in the characteristics themselves-but whether earlier intervention during primary school would narrow the gap is a causal question this design cannot answer.

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Appendices

Appendix A: summary statistics. Appendix B: OLS baseline regressions (age 9 and age 13 predictors). Appendix C: family-structure transitions and father-absence identification. Appendices D–E: variable-level Oaxaca-Blinder decompositions for Maths (gender and father absence). Appendices F–G: corresponding decompositions for English. Appendix H: DFL distributional decompositions (Maths); Appendices I–J: DFL decompositions (English). Appendix K: school-clustering inference sensitivity. Appendix L: level-selection sensitivity in LC Maths. Appendix M: attrition robustness via Lee bounds. Appendix N: additional robustness checks (harmonised wave comparisons, level-mix decompositions, alternative father-absence definitions). Appendix O: attrition balance by gender. Appendix P: LC Maths reporting diagnostics.

A Appendix A. Summary Statistics

Table 13 reports descriptive statistics for the analytical sample (n = 4,333). Leaving Certificate Maths points are harmonised across grading systems and capped at 100. Cognitive skills use standardised logit scores; SDQ subscales range 0–10, with higher scores indicating greater difficulties. The sample comprises cohort members with complete data on the outcome, cognitive assessments, socioemotional indicators, and demographic controls; item-level missingness (not wave-level attrition) is the primary source of exclusion.

Table 14 disaggregates by father-presence status, showing systematic differences in every domain.

Table 13: Summary statistics for the Growing Up in Ireland Child Cohort ('98 cohort), restricted to the analytical sample with valid Leaving Certificate Maths outcomes ($n =$ 4,333). Continuous variables are reported in original units (Maths points or test-score scales), and binary indicators are reported as proportions.

Variable	N	Mean	Std. Dev.	Min	Max
Panel A: Leaving Certificate Performance					
Maths LC Points (Raw)	4,333	60.93	32.84	0.00	125.00
Maths LC Points (Harmonised)	4,333	56.03	27.43	0.00	100.00
New Grading System (post-2017)	4,333	0.39	0.49	0.00	1.00
Panel B: Cognitive Skills					
Wave 1 (Age 9)					
Reading Ability (logit)	4,264	0.35	0.66	-3.36	2.87
Maths Ability (logit)	4,306	-0.48	0.61	-3.62	1.90
Wave 2 (Age 13)					
Verbal Reasoning (logit)	4,105	0.15	0.64	-2.37	1.78
Numerical Ability (logit)	4,093	0.14	0.64	-2.36	2.11
BAS Matrix Reasoning (score)	3,939	119.40	9.48	10.00	161.00
Panel C: Non-Cognitive Skills (SDQ Scales)					
Wave 1 (Age 9)					
Emotional Symptoms	4,330	1.94	1.48	0.00	10.00
Conduct Problems	4,328	1.11	0.99	0.00	9.00
Hyperactivity	4,325	2.73	1.49	0.00	10.00
Peer-relationship Problems	4,322	1.07	0.99	0.00	9.00
Wave 2 (Age 13)					
Emotional Symptoms	4,252	1.65	1.48	0.00	10.00
Conduct Problems	4,252	0.95	0.97	0.00	10.00
Hyperactivity	4,252	2.29	0.98	0.00	10.00
Peer-relationship Problems	4,252	1.01	0.98	0.00	10.00
Panel D: Demographic and Family Characteristics					
Male	4,333	0.48	0.50	0.00	1.00
Wave 1 (Age 9)					
Mother's Education: Upper Secondary	4,333	0.56	0.50	0.00	1.00
Mother's Education: Third Level	4,333	0.32	0.47	0.00	1.00
Father's Education: Upper Secondary	3,808	0.46	0.50	0.00	1.00
Father's Education: Third Level	3,808	0.32	0.47	0.00	1.00
Income Quintile	4,033	3.50	1.47	1.00	5.00
Mixed School	4,051	0.76	0.43	0.00	1.00
Father Missing	4,333	0.12	0.33	0.00	1.00
Wave 2 (Age 13)					
Mother's Education: Upper Secondary	4,253	0.56	0.50	0.00	1.00
Mother's Education: Third Level	4,253	0.36	0.48	0.00	1.00
Father's Education: Upper Secondary	3,435	0.49	0.50	0.00	1.00
Father's Education: Third Level	3,435	0.36	0.48	0.00	1.00
Income Quintile	3,960	3.42	1.47	1.00	5.00
Fee-Paying School	4,124	0.11	0.31	0.00	1.00
DEIS School	4,124	0.11	0.31	0.00	1.00
Mixed School	4,023	0.53	0.50	0.00	1.00
Religious School	4,333	0.67	0.47	0.00	1.00
Father Missing	4,333	0.21	0.41	0.00	1.00
Father Consistently Absent	3,700	0.11	0.31	0.00	1.00

Table 14: Summary statistics by father-presence status in the analytical sample. Columns report group means (father present vs father absent) and mean differences (present minus absent); continuous variables are in original units and binary variables are proportions.

Variable	Father Present	Father Absent	Difference
Panel A: Leaving Certificate Performance (Wave 4)			
Maths LC Points (Raw)	63.72	44.66	19.06***
Maths LC Points (Harmonised)	58.34	42.49	15.85***
Panel B: Cognitive Skills			
Wave 1 (Age 9)			
Reading Ability (logit)	0.27	-0.08	0.35***
Maths Ability (logit)	-0.54	-0.91	0.37***
Wave 2 (Age 13)			
Verbal Reasoning (logit)	0.06	-0.28	0.34***
Numerical Ability (logit)	0.06	-0.33	0.39***
BAS Matrix Reasoning (score)	117.8	112.4	5.38***
Panel C: Socioemotional Skills (SDQ)			
Wave 1 (Age 9)			
Emotional Symptoms	1.90	2.42	-0.52***
Conduct Problems	1.15	1.56	-0.42***
Hyperactivity	2.83	3.57	-0.74***
Peer Problems	1.03	1.52	-0.49***
Wave 2 (Age 13)			
Emotional Symptoms	1.65	2.21	-0.56***
Conduct Problems	0.99	1.39	-0.40***
Hyperactivity	2.40	3.22	-0.82***
Peer Problems	1.03	1.38	-0.36***
Panel D: Demographic and Family Characteristics			
Wave 1 (Age 9)			
Mother's Education (mean)	3.79	3.35	0.44***
Mother's Educ: Upper Secondary (Dummy)	0.57	0.53	0.03**
Mother's Educ: Third Level (Dummy)	0.29	0.19	0.10***
Income Quintile	3.54	2.52	1.02***
Mixed School	0.76	0.73	0.03
Wave 2 (Age 13)			
Mother's Education (mean)	3.96	3.62	0.34***
Mother's Educ: Upper Secondary (Dummy)	0.57	0.57	0.01
Mother's Educ: Third Level (Dummy)	0.33	0.25	0.08***
Income Quintile	3.40	2.72	0.68***
Fee-Paying School	0.11	0.05	0.06***
DEIS School	0.12	0.24	-0.12***
Religious School	0.67	0.43	0.24***
Mixed School	0.53	0.61	-0.08***
Gender (1 = male)	1.50	1.57	-0.07***

Note: Table reports means by father presence status. Differences reflect mean(present) - mean(absent). $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

B Appendix B. OLS Regressions for Leaving Certificate Maths

B.1 Model specifications

I estimate four OLS specifications regressing adjusted Leaving Certificate Maths points on cognitive, socioemotional, socioeconomic, and school covariates. These regressions provide context for the decompositions: endowment components are large only when the underlying predictors strongly predict the outcome.

$$\text{Maths Points}_i = \beta_0 + \sum_k \beta_k \,\text{Cog}_{k,i,w} + \sum_l \beta_l \,\text{SocEmo}_{l,i,w} + \sum_n \beta_n \,\text{SES}_{n,i,w} + \sum_x \beta_x \,\text{School}_{x,i,w} + \epsilon_i \tag{2}$$

Models 1–2 use Wave 1 (age 9) predictors; Models 3–4 use Wave 2 (age 13) predictors. Within each wave, the key variation is the treatment of father's education: odd-numbered models include the full sample with a father's-education-missing indicator, while even-numbered models condition on observed father's education. Table 15 summarises the covariate structure.

All models are estimated with heteroskedasticity-robust standard errors. Sample sizes range from 2,777 to 3,690 depending on wave and father's-education specification. Results are reported in Tables 16 (Wave 1) and 17 (Wave 2).

B.2 Results

Three patterns emerge from Tables 16 and 17. First, Wave 2 models explain substantially more variance ($R^2 \approx 0.39$–0.40) than Wave 1 models ($R^2 \approx 0.30$), confirming that later-measured predictors are more informative for an outcome observed at ages 17–18.

Second, cognitive skills dominate. Numerical ability is the strongest predictor at both ages ($\beta \approx 9.6$ at age 9, rising to 11.7 at age 13; $p < 0.001$), consistent with Duncan et al. (2007). Reading/verbal reasoning contributes an additional 3.6–4.7 points. Among socioemotional traits, only hyperactivity is consistently significant ($\beta \approx -1.5$ to -1.8, $p < 0.001$); conduct problems matter at age 9 but fade by age 13, while peer problems are never significant. Parental education and income show clear gradients at both ages, and missing father's education is associated with lower scores (-6.4 points at age 9, -3.0 at age 13), consistent with paternal disengagement effects. School characteristics-fee-paying (+4.3 points) and DEIS status (-3.3 to -4.8)-become significant predictors at age 13.

Third, the raw gender coefficient shrinks from significant at age 9 ($\beta \approx 3.4$–3.9,

Table 15: Variables included in each OLS specification for Leaving Certificate Maths; checkmarks indicate included covariates and this table reports model design rather than estimated coefficients.

Variable Group	Model 1	Model 2	Model 3	Model 4
Wave	1 (Age 9)	1 (Age 9)	2 (Age 13)	2 (Age 13)
Cognitive Skills				
Numerical Ability	✓	✓	✓	✓
Reading Ability/Verbal Reasoning	✓	✓	✓	✓
BAS Matrices			✓	✓
Socioemotional Traits				
Emotional Symptoms	✓	✓	✓	✓
Conduct Problems	✓	✓	✓	✓
Hyperactivity	✓	✓	✓	✓
Peer-Relationship Problems	✓	✓	✓	✓
Socioeconomic Status				
Mother's Education	✓	✓	✓	✓
Father's Education		✓		✓
Father's Education Missing	✓		✓	
Income Quintile	✓	✓	✓	✓
Individual & School Factors				
Male	✓	✓	✓	✓
CoEd School	✓	✓	✓	✓
Fee Paying School			✓	✓
DEIS School			✓	✓
Religious School			✓	✓

Table 16: Predictors of Leaving Certificate Maths performance from age 9 (Wave 1) covariates. Outcome: adjusted Leaving Certificate Maths points (0–100). Model 1 uses the full Wave 1 covariate sample with a father's-education-missing indicator ($n = 3{,}690$); Model 2 conditions on observed father's education ($n = 3{,}241$). Heteroskedasticity-robust standard errors are reported in parentheses.

Variable	Model 1	Model 2
(Intercept)	47.153***	45.770***
	(1.972)	(2.149)
Numerical Ability	9.637***	9.568***
	(0.587)	(0.621)
Reading Ability	4.125***	3.612***
	(0.555)	(0.588)
Emotional Symptoms	-0.441†	-0.595*
	(0.242)	(0.257)
Conduct Problems	-0.964**	-1.044**
	(0.349)	(0.371)
Hyperactivity	-1.508***	-1.566***
	(0.202)	(0.214)
Peer-relationship Problems	-0.096	0.055
	(0.332)	(0.353)
Mothers Education (Higher Secondary/Technical)	5.887***	4.739**
	(1.352)	(1.507)
Mothers Education (Bachelor's/Postgrad)	12.808***	10.011***
	(1.524)	(1.723)
Fathers Education (Higher Secondary/Technical)	–	5.886***
		(1.167)
Fathers Education (Bachelor's/Postgrad)	–	9.917***
		(1.396)
Income (quintiles, equivalized)	2.459***	1.885***
	(0.352)	(0.385)
Male	3.881***	3.350***
	(0.863)	(0.907)
CoEd	0.957	0.956
	(0.992)	(1.041)
Father's Education Missing	-6.350***	–
	(1.303)	
Observations	3,690	3,241
Residual Std. Error	25.28	24.90
Adjusted R^2	0.297	0.299
F-statistic	130.6***	107.1***

Notes: Standard errors are in parentheses. Significance: $^{\dagger}p < 0.1$, $^{*}p < 0.05$, $^{**}p < 0.01$, $^{***}p < 0.001$. Outcome is adjusted Leaving Certificate Maths points (0–100), corrected for bonus-point overreporting and grading comparability. Parental-education coefficients are relative to the omitted category of at most Lower Secondary education.

Table 17: Predictors of Leaving Certificate Maths performance from age 13 (Wave 2) covariates. Outcome: adjusted Leaving Certificate Maths points (0–100). Model 3 uses the full Wave 2 covariate sample with a father's-education-missing indicator ($n = 3{,}401$); Model 4 conditions on observed father's education ($n = 2{,}777$). Heteroskedasticity-robust standard errors are reported in parentheses.

Variable	Model 3	Model 4
(Intercept)	25.557***	24.032***
	(3.813)	(4.335)
Numerical Ability	11.704***	11.684***
	(0.578)	(0.635)
Verbal Reasoning	4.688***	4.301***
	(0.566)	(0.626)
BAS Matrices Score	0.199***	0.188***
	(0.026)	(0.029)
Emotional Symptoms	-0.446†	-0.706**
	(0.246)	(0.273)
Conduct Problems	-0.249	-0.432
	(0.367)	(0.414)
Hyperactivity	-1.818***	-1.792***
	(0.215)	(0.240)
Peer-relationship Problems	-0.066	-0.114
	(0.317)	(0.352)
Motherś Education (Higher Secondary/Technical)	3.921*	5.003**
	(1.595)	(1.877)
Motherś Education (Bachelor's/Postgrad)	7.371***	7.686***
	(1.722)	(2.031)
Fatherś Education (Higher Secondary/Technical)	–	2.883*
		(1.344)
Fatherś Education (Bachelor's/Postgrad)	–	5.728***
		(1.534)
Income (quintiles, equivalized)	1.823***	1.469***
	(0.324)	(0.372)
Male	0.981	0.834
	(0.837)	(0.916)
Fee-paying School	4.287**	4.228**
	(1.366)	(1.478)
DEIS School	-4.802***	-3.310*
	(1.423)	(1.663)
Religious School	-0.025	0.179
	(1.082)	(1.195)
Mixed School	-2.264*	-1.639
	(0.979)	(1.068)
Father's Education Missing	-2.961**	–
	(1.052)	
Observations	3,401	2,777
Residual Std. Error	23.21	23.01
Adjusted R^2	0.397	0.390
F-statistic	140.7***	105.5***

Notes: Standard errors are in parentheses. Significance: $^{\dagger}p < 0.1$, $^{*}p < 0.05$, $^{**}p < 0.01$, $^{***}p < 0.001$. Outcome is adjusted Leaving Certificate Maths points (0–100), corrected for bonus-point overreporting and grading comparability. Parental-education coefficients are relative to the omitted category of at most Lower Secondary education.

$p < 0.001$) to insignificant at age 13 ($\beta \approx 0.8$–1.0). This is not because the gender gap disappears-the O-B decompositions in the main text show it persists-but because Wave 2 covariates (especially numerical ability) absorb more of the gap through measured endowments, foreshadowing the compositional shift that is the main finding of this paper.

C Appendix C. Family Structure Changes Between Waves

Tables 18–20 trace family-structure transitions across waves and motivate the father-absence proxy used in the main text. Three patterns are especially relevant. First, secondary caregiver (SCG) questionnaire non-completion rises sharply between Waves 2 and 3 (from 5.4% to 11.8% net loss). Second, partnership and marital transitions remain comparatively stable at roughly 3–4% per wave. Third, non-completion increases even among resident SCGs (8.7% to 15.8%), which suggests that survey participation captures engagement differences beyond physical absence.

Table 18: Summary of family-dynamics transitions between Wave 1 and Wave 2 (counts and transition percentages conditional on baseline status).

Transition Type	Count	Percentage
Partner Status Transitions		
No partner → No partner	360	85.1% of initially without partner
No partner → Partner	63	14.9% of initially without partner
Partner → Partner	4,141	96.2% of initially with partner
Partner → No partner	165	3.8% of initially with partner
Primary Caregiver Marital Status Transitions		
Married → Married	3,868	95.9% of initially married
Married → Separated	101	2.5% of initially married
Married → Divorced	33	0.8% of initially married
Separated → Separated	118	60.5% of initially separated
Separated → Married	16	8.2% of initially separated
Separated → Divorced	53	27.2% of initially separated
Never married → Never married	291	81.1% of initially never married
Never married → Married	60	16.7% of initially never married
Secondary Caregiver Participation Transitions		
No SCG → No SCG	360	85.1% of initially without SCG
No SCG → SCG (completed)	38	9.0% of initially without SCG
No SCG → SCG (non-completed)	25	5.9% of initially without SCG
SCG (completed) → SCG (completed)	3,619	87.8% of initial SCG completers
SCG (completed) → SCG (non-completed)	359	8.7% of initial SCG completers
SCG (completed) → No SCG	146	3.5% of initial SCG completers
Key Net Changes		
Net partner loss	102	2.4% of initial partnered households
Net increase in separation/divorce	171	4.2% of initial married households
Net decrease in SCG questionnaire completion	223	5.4% of initial SCG completers

Taken together, these transitions support the interpretation of SCG non-response as a meaningful marker of paternal engagement over time and provide the empirical context for the father-absence decompositions in the main analysis.

Table 19: Summary of family-dynamics transitions between Wave 2 and Wave 3 (counts and transition percentages conditional on baseline status).

Transition Type	Count	Percentage
Partner Status Transitions		
No partner → No partner	480	91.6% of initially without partner
No partner → Partner	44	8.4% of initially without partner
Partner → Partner	3,996	95.5% of initially with partner
Partner → No partner	186	4.5% of initially with partner
Primary Caregiver Marital Status Transitions		
Married → Married	3,771	95.1% of initially married
Married → Separated	114	2.9% of initially married
Married → Divorced	11	0.3% of initially married
Separated → Separated	159	65.1% of initially separated
Separated → Married	9	3.7% of initially separated
Separated → Divorced	55	22.5% of initially separated
Divorced → Divorced	85	56.7% of initially divorced
Divorced → Widowed	53	35.3% of initially divorced
Never married → Never married	267	89.0% of initially never married
Never married → Married	32	10.7% of initially never married
Secondary Caregiver Participation Transitions		
No SCG → No SCG	480	91.6% of initially without SCG
No SCG → SCG (completed)	17	3.2% of initially without SCG
No SCG → SCG (non-completed)	27	5.2% of initially without SCG
SCG (completed) → SCG (completed)	3,009	81.4% of initial SCG completers
SCG (completed) → SCG (non-completed)	584	15.8% of initial SCG completers
SCG (completed) → No SCG	149	4.0% of initial SCG completers
Key Net Changes		
Net partner loss	142	3.4% of initial partnered households
Net increase in separation/divorce	170	4.3% of initial married households
Net decrease in SCG questionnaire completion	436	11.8% of initial SCG completers

Table 20: Comparison of family-dynamics transitions between Waves 1-2 and 2-3 (counts with within-transition percentages in parentheses).

Transition Type	Wave 1 → Wave 2	Wave 2 → Wave 3
Partner Status Transitions		
No partner → No partner	360 (85.1%)	480 (91.6%)
No partner → Partner	63 (14.9%)	44 (8.4%)
Partner → Partner	4,141 (96.2%)	3,996 (95.5%)
Partner → No partner	165 (3.8%)	186 (4.5%)
Primary Caregiver Marital Status Transitions		
Married → Married	3,868 (95.9%)	3,771 (95.1%)
Married → Separated	101 (2.5%)	114 (2.9%)
Married → Divorced	33 (0.8%)	11 (0.3%)
Separated → Separated	118 (60.5%)	159 (65.1%)
Separated → Married	16 (8.2%)	9 (3.7%)
Separated → Divorced	53 (27.2%)	55 (22.5%)
Never married → Never married	291 (81.1%)	267 (89.0%)
Never married → Married	60 (16.7%)	32 (10.7%)
Secondary Caregiver Participation Transitions		
No SCG → No SCG	360 (85.1%)	480 (91.6%)
No SCG → SCG (completed)	38 (9.0%)	17 (3.2%)
No SCG → SCG (non-completed)	25 (5.9%)	27 (5.2%)
SCG (completed) → SCG (completed)	3,619 (87.8%)	3,009 (81.4%)
SCG (completed) → SCG (non-completed)	359 (8.7%)	584 (15.8%)
SCG (completed) → No SCG	146 (3.5%)	149 (4.0%)
Key Net Changes		
Net partner loss	102 (2.4%)	142 (3.4%)
Net increase in separation/divorce	171 (4.2%)	170 (4.3%)
Net decrease in SCG questionnaire completion	223 (5.4%)	436 (11.8%)

Table 21: Wave-3 validation split of the father-absence proxy used in the main analysis. The father-absence indicator is defined as consistent non-response to the father questionnaire in Waves 1 and 2. The table decomposes this group using Wave-3 household partner status and SCG completion, reported within the analytical LC Maths sample.

Group	Wave-3 validation category	n	Share (%)	Mean LC Maths	SD
Father-absent	No resident partner (structural absence)	233	61.2	45.92	34.11
Father-absent	Missing Wave-3 partner status	78	20.5	44.08	33.68
Father-absent	Resident partner without SCG completion (disengaged)	47	12.3	44.06	32.79
Father-absent	Resident partner with SCG completion (engaged)	23	6.0	52.09	35.09
Father-present	Resident partner with SCG completion (engaged)	2040	61.7	64.78	35.39
Father-present	Missing Wave-3 partner status	795	24.1	65.58	37.01
Father-present	Resident partner without SCG completion (disengaged)	380	11.5	59.19	35.10
Father-present	No resident partner (structural absence)	90	2.7	52.21	34.19

Note: Shares are within father-present and father-absent groups, respectively. This split is used to separate structural absence (no resident partner) from resident-partner non-completion of the SCG questionnaire (lower engagement).

D Appendix D. Oaxaca Decompositions: Gender Gaps in Maths, Leaving Cert

This appendix reports the variable-level Oaxaca-Blinder decompositions behind the main-text gender figures. The same pattern shown in the headline plots is visible at the detailed level: with age 9 predictors, the gap is primarily coefficient-led, while with age 13 predictors measured skill differences, especially numeracy, take the larger share.

Table 22: At age 9, the coefficients component drives the gender gap in Maths. Oaxaca-Blinder decomposition of adjusted LC Maths points (0–100) using Wave 1 predictors, with boys' coefficients as reference. "No Father" includes all observations (n = 3,690) with a father's-education-missing indicator; "With Father" conditions on observed father's education (n = 3,241). Negative values indicate components associated with higher male scores. Bootstrap SEs from 1,000 replications.

Statistic	No Father	With Father
Group 1 (Girls)	52.831*** (0.688)	54.749*** (0.717)
Group 2 (Boys)	58.043*** (0.732)	59.183*** (0.729)
Difference	-5.212*** (0.980)	-4.434*** (1.020)
Endowments	-1.570** (0.610)	-1.298** (0.656)
Coefficients	-4.215*** (0.900)	-3.641*** (0.924)
Interaction	0.572 (0.490)	0.505 (0.502)

Variable	Endowments		Coefficients		Interactions	
	No Father	With Father	No Father	With Father	No Father	With Father
Reading Ability	0.021	0.033	0.659*	0.722*	0.014	0.028
	(0.101)	(0.090)	(0.350)	(0.401)	(0.070)	(0.090)
Maths Ability	-1.886***	-1.762***	0.864*	0.809*	0.376*	0.346
	(0.358)	(0.374)	(0.476)	(0.479)	(0.223)	(0.225)
Emotional Symptoms	-0.004	-0.044	-1.343	-1.333	-0.208	-0.191
	(0.109)	(0.096)	(0.856)	(0.902)	(0.151)	(0.143)
Conduct Problems	0.262**	0.327***	1.354	1.595*	-0.194	-0.259
	(0.120)	(0.123)	(0.927)	(0.839)	(0.150)	(0.166)
Hyperactivity	0.784***	0.857***	-1.645	-1.069	0.340	0.224
	(0.195)	(0.191)	(1.298)	(1.331)	(0.275)	(0.287)
Peer-relationship Problems	0.000	0.003	-0.250	-0.320	-0.003	-0.004
	(0.026)	(0.027)	(0.690)	(0.759)	(0.031)	(0.040)
Mothers Educ.	0.007	0.003	0.548	2.069	0.001	0.004
(Higher 2ndary/Tech)	(0.097)	(0.054)	(1.574)	(1.762)	(0.046)	(0.082)
Mothers Educ.	-0.353*	-0.174	0.958	1.211	-0.090	-0.079
(Bachelor's/Postgrad)	(0.194)	(0.133)	(1.106)	(1.320)	(0.133)	(0.115)
Fathers Educ.	-	0.058	-	-0.197	-	-0.004
(Higher 2ndary/Tech)	-	(0.098)	-	(0.990)	-	(0.036)
Fathers Educ.	-	-0.488***	-	-0.246	-	0.035
(Bachelor's/Postgrad)	-	(0.169)	-	(1.011)	-	(0.149)
Father's Educ. Missing	-0.131*	-	-0.140	-	-0.030	-
	(0.072)	-	(0.303)	-	(0.074)	-
Income Quintile	-0.227*	-0.062	0.629	2.067	-0.017	-0.022
	(0.125)	(0.079)	(2.642)	(2.965)	(0.082)	(0.061)
Mixed School	-0.043	-0.049	2.228*	2.359	0.384	0.426
	(0.150)	(0.159)	(1.332)	(1.520)	(0.235)	(0.278)
Constant	–	–	-8.076*	-11.309**	–	–
			(4.210)	(4.881)		

Significance: * p<0.1, ** p<0.05, *** p<0.01. Blinder-Oaxaca decomposition of the gender gap in adjusted LC Maths points using Wave 1 (age 9) predictors. "No Father" uses the full sample (n = 3,690; girls 1,886, boys 1,804) with a father's-education-missing indicator; "With Father" conditions on observed father's education (n = 3,241; girls 1,635, boys 1,606). Endowments capture differences in observed characteristics, coefficients capture differences in returns to those characteristics, and interaction captures their joint contribution. Negative Difference values indicate higher male scores. Bootstrap standard errors from 1,000 replications.

Table 23: By age 13, measured endowments dominate the gender gap in Maths. Oaxaca-Blinder decomposition of adjusted LC Maths points (0–100) using Wave 2 predictors, with boys' coefficients as reference. "No Father" includes all observations with a father's-education-missing indicator; "With Father" conditions on observed father's education. Negative values indicate components associated with higher male scores. Bootstrap SEs from 1,000 replications.

Statistic	No Father	With Father
Group 1 (Girls)	54.209*** (0.699)	56.272*** (0.726)
Group 2 (Boys)	59.091*** (0.732)	60.907*** (0.892)
Difference	-4.882*** (0.998)	-4.635*** (1.150)
Endowments	-4.150*** (0.742)	-4.039*** (0.822)
Coefficients	-1.236 (0.855)	-1.128 (0.963)
Interaction	0.504 (0.531)	0.532 (0.573)

Variable	Endowments		Coefficients		Interactions	
	No Father	With Father	No Father	With Father	No Father	With Father
Verbal Reasoning	-1.034***	-0.803***	-0.389	-0.324	0.275	0.183
	(0.214)	(0.232)	(0.268)	(0.392)	(0.194)	(0.223)
Numerical Ability	-3.658***	-3.546***	-0.151	-0.325	0.146	0.266
	(0.405)	(0.454)	(0.365)	(0.405)	(0.349)	(0.326)
Matrices	-0.186	-0.259	-9.436*	-13.013*	0.061	0.115
	(0.152)	(0.172)	(5.485)	(7.647)	(0.072)	(0.109)
Emotional Symptoms	-0.006	-0.083	-1.130	-1.143	-0.269	-0.299
	(0.124)	(0.161)	(0.692)	(0.797)	(0.182)	(0.237)
Conduct Problems	0.012	0.023	0.316	0.219	-0.009	-0.009
	(0.041)	(0.055)	(0.801)	(0.795)	(0.048)	(0.051)
Hyperactivity	0.970***	0.948***	-1.636	-1.303	0.401	0.322
	(0.229)	(0.222)	(1.280)	(1.189)	(0.323)	(0.296)
Peer-relationship Problems	0.080	0.059	1.124	0.601	-0.144	-0.083
	(0.064)	(0.067)	(0.765)	(0.712)	(0.113)	(0.095)
Mothers Educ.	0.099	0.100	-1.010	1.211	-0.038	0.056
(Higher 2ndary/Tech)	(0.094)	(0.133)	(1.552)	(2.100)	(0.079)	(0.141)
Mothers Educ.	-0.217	-0.179	0.863	1.847	-0.076	-0.151
(Bachelor's/Postgrad)	(0.137)	(0.161)	(1.295)	(1.617)	(0.143)	(0.183)
Fathers Educ.	-	0.081	-	0.570	-	0.045
(Higher 2ndary/Tech)	-	(0.093)	-	(1.224)	-	(0.101)
Fathers Educ.	-	-0.277*	-	0.485	-	-0.068
(Bachelor's/Postgrad)	-	(0.148)	-	(1.223)	-	(0.183)
Father Educ. Missing	-0.123	-	0.101	-	0.022	-
	(0.082)	-	(0.360)	-	(0.084)	-
Income Quintile	-0.063	-0.002	0.635	1.532	-0.007	-0.001
	(0.088)	(0.065)	(2.018)	(2.601)	(0.034)	(0.044)
Fee Paying School	-0.126	-0.142*	0.048	-0.100	-0.012	0.024
	(0.083)	(0.069)	(0.322)	(0.315)	(0.088)	(0.076)
Other School Variables†	–	–	–	–	–	–
Constant	–	–	10.688	9.830	–	–
			(7.190)	(8.819)		

Significance: * p<0.1, ** p<0.05, *** p<0.01. Blinder-Oaxaca decomposition of the gender gap in adjusted LC Maths points using Wave 2 (age 13) predictors. "No Father" uses the full sample (n = 3,401; girls 1,724, boys 1,677) with a father's-education-missing indicator; "With Father" conditions on observed father's education (n = 2,777; girls 1,377, boys 1,400). Endowments capture differences in observed characteristics, coefficients capture differences in returns to those characteristics, and interaction captures their joint contribution. Negative Difference values indicate higher male scores. Bootstrap standard errors from 1,000 replications. † Other school variables were included but are not shown because they were not statistically significant.

E Appendix E. Oaxaca Decompositions: Father Absence Effects in Maths, Leaving Cert

The detailed decomposition results underlying the figures on father absence effects in the main text are reported in the tables below.

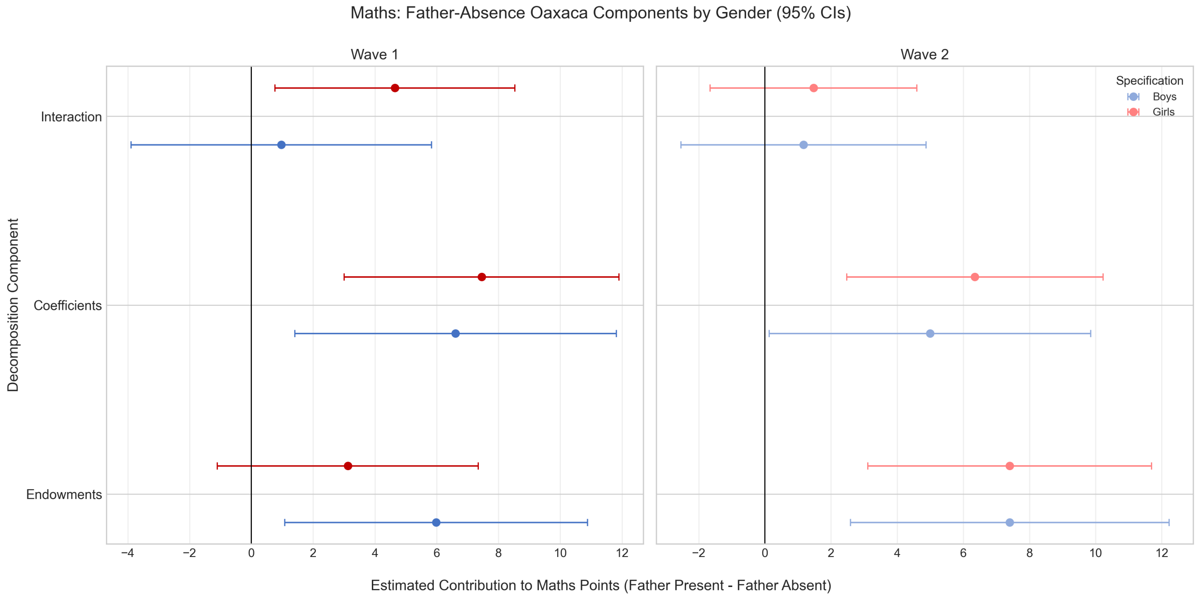

Figure 5: Coefficient-plot representation of Oaxaca-Blinder components for father-absence gaps in Leaving Certificate Maths, estimated separately by gender. Points denote component estimates (endowments, coefficients, interaction) and horizontal whiskers denote 95% confidence intervals based on bootstrap standard errors (1,000 replications). Panels separate Wave 1 (age 9 predictors) and Wave 2 (age 13 predictors). Positive values indicate components associated with higher scores for the father-present group relative to the father-absent group. Estimates are descriptive and should not be interpreted as causal effects.

Figure 5 complements Tables 24 and 25 by presenting the same components as point estimates with 95% confidence intervals, making the cross-wave and cross-gender uncertainty comparisons more transparent.

Table 24: At age 9, the father-absence penalty in Maths operates through both endowments and coefficients. Oaxaca-Blinder decomposition of adjusted LC Maths points (0–100) using Wave 1 predictors, estimated separately for boys ($n_{\text{present}} = 1{,}188$, $n_{\text{absent}} = 126$) and girls ($n_{\text{present}} = 1{,}142$, $n_{\text{absent}} = 150$). Father-present coefficients as reference. Positive values indicate higher scores for father-present students. Bootstrap SEs from 1,000 replications.

Statistic	Boys	Girls
Group 1 (Father Present)	60.834*** (0.892)	55.799*** (0.846)
Group 2 (Father Absent)	47.270*** (2.759)	40.573*** (2.247)
Difference	13.564*** (2.921)	15.225*** (2.415)
Endowments	5.984** (2.502)	3.122 (2.156)
Coefficients	6.612** (2.655)	7.456*** (2.270)
Interaction	0.969 (2.483)	4.647** (1.982)

Variable	Endowments		Coefficients		Interactions	
	Boys	Girls	Boys	Girls	Boys	Girls
Reading Ability	0.842	0.294	-0.285	-0.378	-0.238	-0.058
	(0.790)	(0.572)	(1.078)	(1.033)	(0.771)	(0.303)
Maths Ability	2.807**	1.326	0.193	-1.425	-0.083	0.391
	(1.271)	(0.900)	(2.410)	(2.397)	(1.053)	(0.639)
Emotional Symptoms	-0.929	0.252	-3.883	-2.351	0.837	0.513
	(0.858)	(0.561)	(3.164)	(2.583)	(0.862)	(0.634)
Conduct Problems	-0.379	-0.048	-5.076*	-0.156	0.872	0.037
	(0.557)	(0.537)	(2.925)	(2.316)	(0.744)	(0.562)
Hyperactivity	1.177	0.634	-0.081	-2.602	0.020	0.557
	(1.221)	(0.651)	(4.776)	(3.287)	(1.268)	(0.788)
Peer-relationship Problems	1.049	0.001	3.358	0.528	-1.181	-0.123
	(0.794)	(0.424)	(2.399)	(2.066)	(0.898)	(0.470)
Mothers Educ. (Higher 2ndary/Tech)	0.569	-0.183	-9.889***	5.997*	-0.414	0.527
	(1.041)	(0.358)	(3.570)	(3.074)	(0.771)	(0.625)
Mothers Educ. (Bachelor's/Postgrad)	0.944	1.290	-3.294	-1.028	-0.424	-0.283
	(1.111)	(1.154)	(2.411)	(1.879)	(0.661)	(0.809)
Income Quintile	-0.038	-0.616	6.929	9.781*	1.523	3.180*
	(1.422)	(1.639)	(6.580)	(5.141)	(1.589)	(1.810)
Mixed School	-0.058	0.172	-4.745	-1.262	0.057	-0.092
	(0.407)	(0.292)	(4.288)	(3.969)	(0.423)	(0.306)
Constant	–	–	23.384**	0.352	–	–
			(9.465)	(8.197)		

Significance: * p<0.1, ** p<0.05, *** p<0.01. Blinder-Oaxaca decomposition of father-absence differences in adjusted LC Maths points using Wave 1 (age 9) predictors, estimated separately for boys ($n = 1{,}314$; present 1,188, absent 126) and girls ($n = 1{,}292$; present 1,142, absent 150). Endowments capture observed-characteristic differences between father-present and father-absent students; coefficients capture differential returns; interaction captures their joint contribution. Positive Difference values indicate higher scores among father-present students. Bootstrap standard errors from 1,000 replications.

Table 25: By age 13, endowments account for a larger share of the father-absence penalty in Maths. Oaxaca-Blinder decomposition of adjusted LC Maths points (0–100) using Wave 2 predictors, estimated separately by gender. Father-present coefficients as reference. Positive values indicate higher scores for father-present students. Bootstrap SEs from 1,000 replications.

Statistic	Boys	Girls
Group 1 (Father Present)	60.834*** (0.844)	55.799*** (0.850)
Group 2 (Father Absent)	47.270*** (2.752)	40.573*** (2.370)
Difference	13.564*** (2.875)	15.225*** (2.520)
Endowments	7.406*** (2.456)	7.405*** (2.189)
Coefficients	4.992** (2.482)	6.351*** (1.977)
Interaction	1.166 (1.891)	1.469 (1.594)

Variable	Endowments		Coefficients		Interactions	
	Boys	Girls	Boys	Girls	Boys	Girls
Verbal Reasoning	1.410	1.165*	-0.945	0.169	-0.742	-0.396
	(1.008)	(0.639)	(0.989)	(0.326)	(0.651)	(0.576)
Numerical Ability	2.020	2.940**	0.372	-0.428	1.490	0.511
	(1.552)	(1.266)	(0.710)	(0.941)	(1.330)	(1.114)
BAS Matrices	0.880	0.638	3.991	-7.021	0.151	-0.177
	(0.699)	(0.408)	(18.694)	(13.469)	(0.698)	(0.359)
Emotional Symptoms	-0.608	0.160	-2.156	-2.202	0.593	0.440
	(0.662)	(0.502)	(2.553)	(2.518)	(0.745)	(0.609)
Conduct Problems	-0.046	0.074	-1.420	0.226	0.108	-0.037
	(0.244)	(0.318)	(1.767)	(1.897)	(0.342)	(0.326)
Hyperactivity	1.504	1.458*	-0.336	3.418	0.099	-0.658
	(0.921)	(0.825)	(3.288)	(3.294)	(0.986)	(0.693)
Peer-relationship Problems	0.861	-0.851	3.349	-2.479	-0.767	0.752
	(0.666)	(0.595)	(2.365)	(1.872)	(0.708)	(0.607)
Mothers Educ. (Higher 2ndary/Tech)	0.411	-0.114	-4.969	8.428***	-0.277	0.191
	(0.767)	(0.508)	(3.491)	(3.120)	(0.573)	(0.790)
Mothers Educ. (Bachelor's/Postgrad)	0.235	0.491	0.001	1.454	0.000	0.376
	(0.502)	(0.818)	(3.044)	(2.655)	(0.403)	(0.799)
Income Quintile	0.967	0.151	-3.413	5.468	-0.421	1.183
	(0.706)	(1.047)	(5.682)	(5.425)	(0.702)	(1.126)
Fee Paying School	-1.065*	0.371	1.884**	-0.210	1.246*	-0.176
	(0.619)	(0.756)	(0.769)	(0.519)	(0.695)	(0.740)
Constant	–	–	14.486	-2.961	–	–
			(21.513)	(19.086)		

Significance: * p<0.1, ** p<0.05, *** p<0.01. Blinder-Oaxaca decomposition of father-absence differences in adjusted LC Maths points using Wave 2 (age 13) predictors, estimated separately for boys (n = 1,314) and girls (n = 1,292). Endowments capture observed-characteristic differences between father-present and father-absent students; coefficients capture differential returns; interaction captures their joint contribution. Positive Difference values indicate higher scores among father-present students. Bootstrap standard errors from 1,000 replications. Contributions from other school control variables (DEIS School, Mixed School, Religious School) included in the model are not itemized separately but are incorporated into the Endowments, Coefficients, and Interaction totals. This explains why the sum of the itemized rows does not exactly match the component totals.

F Appendix F. Oaxaca Decompositions: Gender Gaps in English, Leaving Cert

This appendix reports the variable-level Oaxaca-Blinder decompositions for the English gender gap (Tables 26 and 27), which provide the cross-subject comparison discussed in Section 3.4.

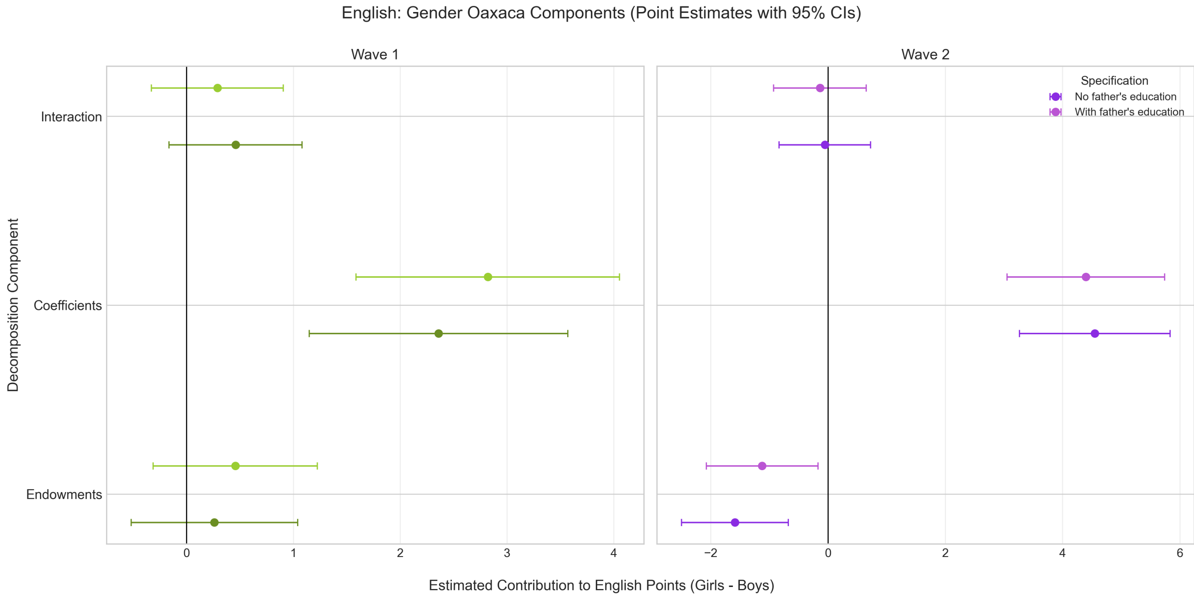

Figure 6: Coefficient-plot representation of Oaxaca-Blinder components for the gender gap in Leaving Certificate English. Points denote component estimates (endowments, coefficients, interaction) and horizontal whiskers denote 95% confidence intervals based on bootstrap standard errors (1,000 replications). Panels separate Wave 1 (age 9 predictors) and Wave 2 (age 13 predictors), each shown with and without father's education controls. Positive values indicate components associated with higher female scores. Estimates are descriptive and should not be interpreted as causal effects.

Figure 6 presents the same decomposition quantities as the appendix tables in coefficient-plot form, which facilitates direct inspection of uncertainty around the reversal pattern between endowments and coefficients in English.

Across models, the decomposition results show that girls score higher than boys in English at the Leaving Certificate, with an average gap of about 3 points across both the Wave 1 and Wave 2 models. Most of this gap comes from the coefficients component-meaning that, for the same set of observed characteristics, the mapping into outcomes differs between boys and girls. Differences in measured skill levels themselves explain a smaller part of the gap. This suggests that the gender gap in English is coefficient-dominant rather than endowment-dominant. The patterns stay similar whether or not paternal education is included in the models.

Table 26: In English, girls' advantage is driven by coefficients rather than endowments (age 9). Oaxaca-Blinder decomposition of adjusted LC English points using Wave 1 predictors, with boys' coefficients as reference. Positive values indicate components associated with higher female scores. This pattern reverses the Maths decomposition, where endowments dominate by age 13. Bootstrap SEs from 1,000 replications.

Statistic	No Father	With Father
Group 1 (Girls)	69.065*** (0.434)	69.875*** (0.444)
Group 2 (Boys)	65.986*** (0.492)	66.309*** (0.497)
Difference	3.079*** (0.649)	3.566*** (0.677)
Endowments	0.260 (0.398)	0.456 (0.393)
Coefficients	2.360*** (0.618)	2.821*** (0.630)
Interaction	0.459 (0.318)	0.288 (0.315)

Variable	Endowments		Coefficients		Interactions	
	No Father	With Father	No Father	With Father	No Father	With Father
Reading Ability	0.079	0.146	0.015	-0.056	0.001	-0.003
	(0.223)	(0.219)	(0.260)	(0.268)	(0.024)	(0.027)
Maths Ability	-0.190*	-0.147	0.173	0.129	0.074	0.053
	(0.112)	(0.101)	(0.315)	(0.291)	(0.132)	(0.122)
Emotional Symptoms	0.034	0.063	0.146	-0.266	0.022	-0.038
	(0.077)	(0.075)	(0.627)	(0.657)	(0.096)	(0.096)
Conduct Problems	0.135**	0.178**	0.290	0.614	-0.044	-0.104
	(0.068)	(0.084)	(0.566)	(0.602)	(0.089)	(0.106)
Hyperactivity	0.688***	0.705***	-0.207	-0.051	0.043	0.011
	(0.158)	(0.173)	(0.853)	(0.946)	(0.178)	(0.208)
Peer-relationship Problems	0.000	-0.001	-0.510	-0.352	-0.010	-0.006
	(0.017)	(0.018)	(0.527)	(0.557)	(0.033)	(0.038)
Mothers Educ.	0.004	0.001	0.947	0.911	0.002	0.000
(Higher 2ndary/Tech)	(0.055)	(0.055)	(1.237)	(1.258)	(0.050)	(0.052)
Mothers Educ.	-0.196*	-0.098	0.703	0.751	-0.065	-0.045
(Bachelor's/Postgrad)	(0.113)	(0.087)	(0.791)	(0.816)	(0.095)	(0.079)
Fathers Educ.	-	0.019	-	0.417	-	0.006
(Higher 2ndary/Tech)	-	(0.059)	-	(0.724)	-	(0.037)
Fathers Educ.	-	-0.178*	-	0.635	-	-0.087
(Bachelor's/Postgrad)	-	(0.106)	-	(0.664)	-	(0.097)
Father's Educ. Missing	-0.019	-	-0.166	-	-0.043	-
	(0.044)	-	(0.218)	-	(0.062)	-
Income Quintile	-0.070	-0.013	0.803	1.072	-0.024	-0.012
	(0.054)	(0.029)	(1.885)	(1.871)	(0.060)	(0.036)
Mixed School	-0.205*	-0.217**	2.944**	2.803**	0.502***	0.512***
	(0.115)	(0.110)	(0.972)	(0.917)	(0.181)	(0.176)
Constant	–	–	-2.777	-3.785	–	–
			(2.658)	(2.778)		

Significance: * p<0.1, ** p<0.05, *** p<0.01. Blinder-Oaxaca decomposition of the gender gap in adjusted LC English points using Wave 1 (age 9) predictors. "No Father" uses the full sample (n = 3,679; girls 1,887, boys 1,792) with a father's-education-missing indicator; "With Father" conditions on observed father's education (n = 3,233; girls 1,633, boys 1,600). Endowments capture differences in observed characteristics, coefficients capture differences in returns to those characteristics, and interaction captures their joint contribution. Positive Difference values indicate higher female scores. Bootstrap standard errors from 1,000 replications.

Table 27: At age 13, the coefficient-dominant pattern in English persists: the coefficients component more than offsets boys' endowment advantage. Oaxaca-Blinder decomposition of adjusted LC English points using Wave 2 predictors, with boys' coefficients as reference. Bootstrap SEs from 1,000 replications.

Statistic	No Father	With Father
Group 1 (Girls)	69.557*** (0.483)	70.585*** (0.501)
Group 2 (Boys)	66.650*** (0.457)	67.450*** (0.630)
Difference	2.906*** (0.678)	3.135*** (0.726)
Endowments	-1.588*** (0.465)	-1.123** (0.486)
Coefficients	4.548*** (0.655)	4.396*** (0.685)
Interaction	-0.054 (0.399)	-0.138 (0.403)

Variable	Endowments		Coefficients		Interactions	
	No Father	With Father	No Father	With Father	No Father	With Father
Verbal Reasoning	-1.655***	-1.363***	-0.411	-0.577*	0.282*	0.312*
	(0.253)	(0.290)	(0.236)	(0.303)	(0.163)	(0.185)
Numerical Ability	-0.553***	-0.420**	0.235	0.420	-0.228	-0.343
	(0.211)	(0.193)	(0.288)	(0.309)	(0.281)	(0.259)
Matrices	-0.026	-0.041	-6.556	-7.606	0.047	0.061
	(0.035)	(0.046)	(4.467)	(5.375)	(0.050)	(0.068)
Emotional Symptoms	-0.013	0.016	-0.289	-0.388	-0.069	-0.105
	(0.089)	(0.110)	(0.421)	(0.537)	(0.103)	(0.149)
Conduct Problems	-0.011	-0.003	-0.693	-0.539	0.022	0.020
	(0.025)	(0.027)	(0.575)	(0.555)	(0.042)	(0.042)
Hyperactivity	0.677***	0.720***	-0.107	0.008	0.026	-0.002
	(0.169)	(0.173)	(0.931)	(0.888)	(0.226)	(0.215)
Peer-relationship Problems	0.043	0.026	-0.115	-0.298	0.014	0.038
	(0.044)	(0.044)	(0.496)	(0.525)	(0.065)	(0.075)
Mothers Educ.	0.082	0.086	-1.196	-0.854	-0.037	-0.036
(Higher 2ndary/Tech)	(0.089)	(0.077)	(1.507)	(1.680)	(0.091)	(0.090)
Mothers Educ.	-0.178	-0.124	-0.614	-0.240	0.052	0.018
(Bachelor's/Postgrad)	(0.119)	(0.096)	(1.136)	(1.313)	(0.115)	(0.115)
Fathers Educ.	"	0.048	"	1.001	"	0.082
(Higher 2ndary/Tech)	"	(0.056)	"	(0.910)	"	(0.095)
Fathers Educ.	"	-0.111	"	1.051	"	-0.153
(Bachelor's/Postgrad)	"	(0.089)	"	(0.900)	"	(0.136)
Father Educ. Missing	-0.053	"	0.121	"	0.030	"
	(0.053)	"	(0.246)	"	(0.061)	"
Income Quintile	-0.027	-0.002	1.928	2.205	-0.022	-0.003
	(0.042)	(0.024)	(1.470)	(2.148)	(0.035)	(0.049)
Mixed School	0.136*	0.075	1.406*	0.828	-0.159	-0.076
	(0.071)	(0.064)	(0.737)	(0.819)	(0.102)	(0.085)
Constant	"	"	9.425	8.364	"	"
			(5.933)	(6.727)		

Significance: * p<0.1, ** p<0.05, *** p<0.01. Blinder-Oaxaca decomposition of the gender gap in adjusted LC English points using Wave 2 (age 13) predictors. "No Father" uses the full sample (n = 3,375; girls 1,719, boys 1,656) with a father's-education-missing indicator; "With Father" conditions on observed father's education (n = 2,756; girls 1,370, boys 1,386). Endowments capture differences in observed characteristics, coefficients capture differences in returns to those characteristics, and interaction captures their joint contribution. Positive Difference values indicate higher female scores. Bootstrap standard errors from 1,000 replications.

G Appendix G. Oaxaca Decompositions: Father Absence Effects in English, Leaving Cert

This appendix reports Oaxaca-Blinder decompositions for father-absence gaps in English, complementing the Maths results in Appendix E. The broad pattern is similar across subjects: father absence is associated with lower achievement for both boys and girls, and much of that association is captured by endowment differences, especially cognitive traits such as verbal reasoning. Coefficient effects are smaller and less stable, and interaction terms are generally imprecise. This is consistent with the interpretation in the main text that measured-skill differences account for most of the father-absence penalty.

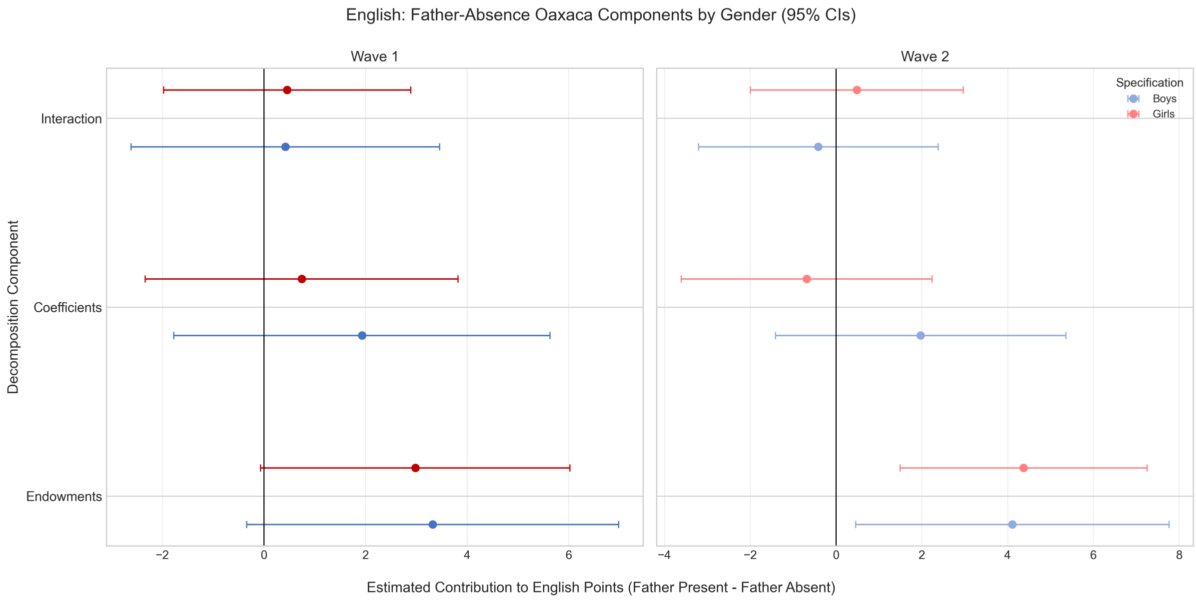

Figure 7: Coefficient-plot representation of Oaxaca-Blinder components for father-absence gaps in Leaving Certificate English, estimated separately by gender. Points denote component estimates (endowments, coefficients, interaction) and horizontal whiskers denote 95% confidence intervals based on bootstrap standard errors (1,000 replications). Panels separate Wave 1 (age 9 predictors) and Wave 2 (age 13 predictors). Positive values indicate components associated with higher scores for the father-present group relative to the father-absent group. Estimates are descriptive and should not be interpreted as causal effects.

Figure 7 provides a compact visual summary of the English father-absence decompositions in Tables 28 and 29, with confidence intervals that highlight which component differences are estimated with greater precision.

Table 28: Father-absence penalties in English are smaller than in Maths and are endowment-driven for both genders at age 9. Oaxaca-Blinder decomposition of adjusted LC English points using Wave 1 predictors, estimated separately for boys ($n_{\text{present}} = 1{,}178$, $n_{\text{absent}} = 122$) and girls ($n_{\text{present}} = 1{,}138$, $n_{\text{absent}} = 151$). Father-present coefficients as reference. Bootstrap SEs from 1,000 replications.

Statistic	Boys	Girls
Group 1 (Father Present)	66.911*** (0.568)	70.569*** (0.544)
Group 2 (Father Absent)	61.238*** (2.023)	66.391*** (1.634)
Difference	5.673*** (2.105)	4.179** (1.707)
Endowments	3.323* (1.869)	2.980* (1.554)
Coefficients	1.930 (1.890)	0.742 (1.572)
Interaction	0.420 (1.551)	0.457 (1.242)

Variable	Endowments		Coefficients		Interactions	
	Boys	Girls	Boys	Girls	Boys	Girls
Reading Ability	1.670	0.622	-0.278	-0.198	-0.243	-0.049
	(1.067)	(0.700)	(0.490)	(0.616)	(0.592)	(0.223)
Maths Ability	0.222	0.131	0.273	0.002	-0.110	-0.000
	(0.597)	(0.354)	(1.403)	(1.361)	(0.613)	(0.387)
Emotional Symptoms	0.115	-0.457	0.833	-2.162	-0.181	0.486
	(0.528)	(0.460)	(2.463)	(2.219)	(0.560)	(0.517)
Conduct Problems	-0.266	0.015	-2.634	0.732	0.484	-0.167
	(0.384)	(0.393)	(1.730)	(1.799)	(0.466)	(0.437)
Hyperactivity	0.707	0.729	-1.826	-1.135	0.465	0.235
	(0.810)	(0.490)	(3.087)	(2.232)	(0.814)	(0.544)
Peer-relationship Problems	-0.192	0.335	-0.795	0.794	0.282	-0.197
	(0.614)	(0.348)	(1.746)	(1.438)	(0.651)	(0.360)
Mothers Educ.	0.263	0.044	-2.557	2.145	-0.140	0.139
(Higher 2ndary/Tech)	(0.585)	(0.280)	(3.364)	(2.431)	(0.493)	(0.315)
Mothers Educ.	0.833	0.222	-4.867**	1.362	-0.598	0.398
(Bachelor's/Postgrad)	(1.065)	(0.366)	(2.302)	(1.270)	(0.820)	(0.477)
Income Quintile	-0.073	1.148	2.067	-0.758	0.457	-0.256
	(0.985)	(0.939)	(4.838)	(2.900)	(1.060)	(0.951)
Mixed School	0.044	0.192	-0.126	-2.134	0.005	-0.131
	(0.218)	(0.250)	(3.290)	(3.107)	(0.220)	(0.253)
Constant	–	–	11.839	2.095	–	–
			(7.898)	(6.873)		

Significance: * p<0.1, ** p<0.05, *** p<0.01. Blinder-Oaxaca decomposition of father-absence differences in adjusted LC English points using Wave 1 (age 9) predictors, estimated separately for boys ($n = 1{,}300$; present 1,178, absent 122) and girls ($n = 1{,}289$; present 1,138, absent 151). Endowments capture observed-characteristic differences between father-present and father-absent students; coefficients capture differential returns; interaction captures their joint contribution. Positive Difference values indicate higher scores among father-present students. Bootstrap standard errors from 1,000 replications.

Table 29: At age 13, endowments account for the entire father-absence penalty in English, particularly for girls where the composition effect exceeds the total gap. Oaxaca-Blinder decomposition of adjusted LC English points using Wave 2 predictors, estimated separately for boys ($n_{\text{present}} = 1{,}178$, $n_{\text{absent}} = 122$) and girls ($n_{\text{present}} = 1{,}138$, $n_{\text{absent}} = 151$). Father-present coefficients as reference. Bootstrap SEs from 1,000 replications.

Statistic	Boys	Girls
Group 1 (Father Present)	66.911*** (0.585)	70.569*** (0.539)
Group 2 (Father Absent)	61.238*** (1.944)	66.391*** (1.591)
Difference	5.673*** (2.036)	4.179** (1.672)
Endowments	4.112** (1.864)	4.376*** (1.469)
Coefficients	1.975 (1.728)	-0.684 (1.492)
Interaction	-0.413 (1.425)	0.487 (1.267)

Variable	Endowments		Coefficients		Interactions	
	Boys	Girls	Boys	Girls	Boys	Girls
Verbal Reasoning	1.450	1.918***	-0.925	0.207	-0.534	-0.524
	(1.212)	(0.706)	(0.800)	(0.251)	(0.616)	(0.430)
Numerical Ability	-1.031	1.056	0.451	0.257	1.330	-0.308
	(0.935)	(0.709)	(0.500)	(0.538)	(0.951)	(0.647)
BAS Matrices	0.488	-0.108	-6.891	0.384	-0.231	0.012
	(0.504)	(0.468)	(11.437)	(13.959)	(0.466)	(0.469)
Emotional Symptoms	0.169	0.209	0.839	0.534	-0.248	-0.095
	(0.515)	(0.455)	(1.898)	(2.185)	(0.572)	(0.453)
Conduct Problems	-0.107	-0.010	-0.822	-0.126	0.104	0.018
	(0.367)	(0.242)	(2.007)	(1.454)	(0.373)	(0.249)
Hyperactivity	1.218	0.435	-0.627	-0.225	0.191	0.039
	(0.892)	(0.516)	(2.953)	(2.693)	(0.903)	(0.519)
Peer-relationship Problems	-0.199	-0.035	-1.076	-0.793	0.251	0.237
	(0.304)	(0.592)	(1.360)	(1.935)	(0.355)	(0.653)
Mothers Educ. (Higher 2ndary/Tech)	0.142	-0.031	-0.221	4.278	-0.012	0.056
	(0.410)	(0.257)	(3.385)	(3.095)	(0.355)	(0.359)
Mothers Educ. (Bachelor's/Postgrad)	0.385	-0.338	-2.110	2.766*	-0.199	0.768
	(0.718)	(0.562)	(2.641)	(1.526)	(0.550)	(0.764)
Income Quintile	0.138	0.673	-0.096	0.408	-0.011	0.085
	(0.405)	(0.733)	(3.818)	(3.301)	(0.468)	(0.707)
Fee Paying School	-0.201	0.073	0.342	-0.047	0.213	-0.038
	(0.451)	(0.370)	(0.663)	(0.239)	(0.463)	(0.380)
DEIS School	1.665**	0.735	2.334*	0.205	-1.299*	-0.102
	(0.806)	(0.552)	(1.246)	(0.913)	(0.765)	(0.476)
Mixed School	0.045	0.041	-0.227	0.661	0.013	-0.061
	(0.223)	(0.273)	(2.581)	(2.080)	(0.244)	(0.273)
Religious School	-0.049	-0.243	0.781	2.423	0.020	0.399
	(0.240)	(0.671)	(3.206)	(3.130)	(0.238)	(0.717)
Constant	–	–	10.223	-11.616	–	–
			(14.736)	(18.561)		

Significance: * p<0.1, ** p<0.05, *** p<0.01. Blinder-Oaxaca decomposition of father-absence differences in adjusted LC English points using Wave 2 (age 13) predictors, estimated separately for boys ($n = 1{,}300$; present 1,178, absent 122) and girls ($n = 1{,}289$; present 1,138, absent 151). Endowments capture observed-characteristic differences between father-present and father-absent students; coefficients capture differential returns; interaction captures their joint contribution. Positive Difference values indicate higher scores among father-present students. Bootstrap standard errors from 1,000 replications.

H Appendix H. Distributional Decomposition of Gender Gaps in Maths Achievement

While the main analysis employs Oaxaca-Blinder decompositions to examine mean differences, this approach cannot capture how gaps vary across the achievement distribution. I complement it with DiNardo-Fortin-Lemieux (DiNardo et al., 1996) (DFL) decompositions, which allow examination of gender and father-absence gaps at specific quantiles (0.10, 0.25, 0.50, 0.75, 0.90), do not impose the linearity assumptions required by Oaxaca-Blinder, and reveal whether composition and structure effects differ for low-, medium-, and high-achieving students.

Choice of DFL over RIF

An alternative distributional decomposition approach is the recentered influence function (RIF) regression method of Firpo et al. (2009). The RIF for the τ-th quantile requires estimating $f_Y(q_\tau)$, the density of the outcome evaluated at the quantile, via kernel density estimation. Leaving Certificate Maths points are a bounded sum of discrete grade bands across six subjects (H1=100, H2=88, ...), creating a distribution with pronounced mass points where this density is not well defined in the continuous sense. Fortin et al. (2011) note explicitly that their distributional methods work best for "continuous and unbounded" outcome variables, and Rios-Avila (2020) demonstrate that the linear approximation underlying RIF regressions introduces specification error that compounds when the outcome has limited support. The DFL reweighting approach avoids this problem entirely: it operates on the empirical distribution function through propensity score reweighting and does not require density estimation at specific quantile points. For the same reason, DFL is better suited to the small father-absence cells (N≈126–150 per gender), where kernel density estimation would be particularly unreliable. I therefore adopt DFL as the primary distributional decomposition method.

Inference is based on 1,000 stratified bootstrap replications (resampling separately within each group to preserve group sizes), yielding pointwise 95% percentile confidence intervals for the total gap, composition effect, and structure effect at each quantile.

H.1 Decomposition of Gender Differences

The main DFL results for the gender gap in Maths are presented in Section 3.5 (Figure 4 and Table 3). This appendix expands on the estimation details and the density-based interpretation.

Following DiNardo et al. (1996), I construct a counterfactual distribution by reweighting girls' Maths scores with propensity weights from a logistic model of gender on observables.

Counterfactual quantiles are then computed directly as weighted quantiles of the girls' outcome distribution, which avoids extra simulation noise from random draws.

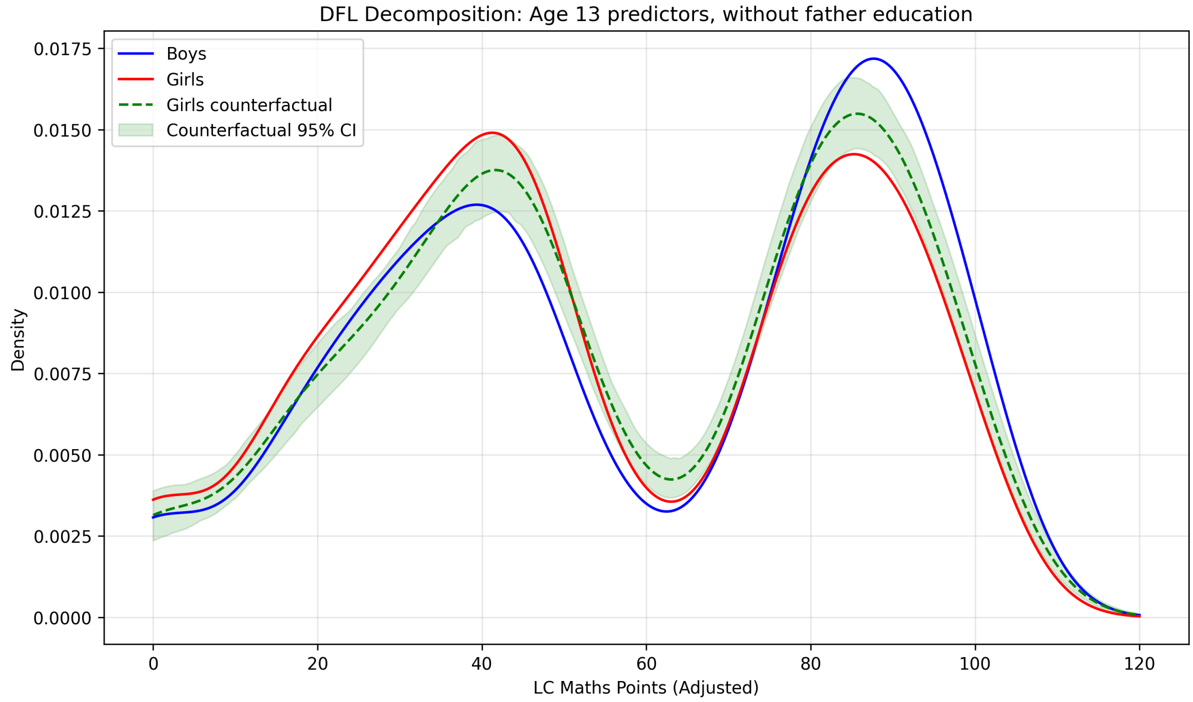

Figure 8: DFL density decomposition: age 13 predictors, without father's education. Maths score distributions for boys (blue), girls (red), and the counterfactual distribution (green dashed, with 95% bootstrap confidence band in shaded green). The counterfactual shows what girls' scores would look like if they had the same observable characteristics as boys. The bimodal shape reflects the Ordinary/Higher Level split in LC Maths. The composition effect (distance between red and green lines) is most visible in the middle of the distribution.

The distance between girls' observed scores (red) and the counterfactual (green) is the composition effect, while the distance between the counterfactual and boys' scores (blue) is the structure effect. In practice, both channels contribute, but composition becomes more visible around the middle of the distribution.

At lower quantiles (0.1–0.3), reweighting girls to boys' observables changes outcomes little and can even reduce predicted scores in some specifications. By contrast, at middle and upper quantiles, the composition effect becomes larger, indicating that observable characteristics explain more of the gap for middle and higher achievers.

Bootstrap intervals show that many quantile-specific composition estimates are imprecise, especially in the tails where effective cell sizes are smaller. At the 0.75 and 0.90 quantiles, composition is often statistically indistinguishable from zero, so structure effects and unobserved differences play a larger role there. Figure 8 also makes the bimodal LC Maths distribution explicit, consistent with the Ordinary/Higher level split, and shows the counterfactual lying between the male and female distributions.

H.2 Decomposition of Father Absence Effects

I then apply the same DFL framework to father-absence gaps, estimated separately by gender with predictors from age 9 (Wave 1) and age 13 (Wave 2). The counterfactual asks what father-present students would score if they had the observable profile of father-absent students.

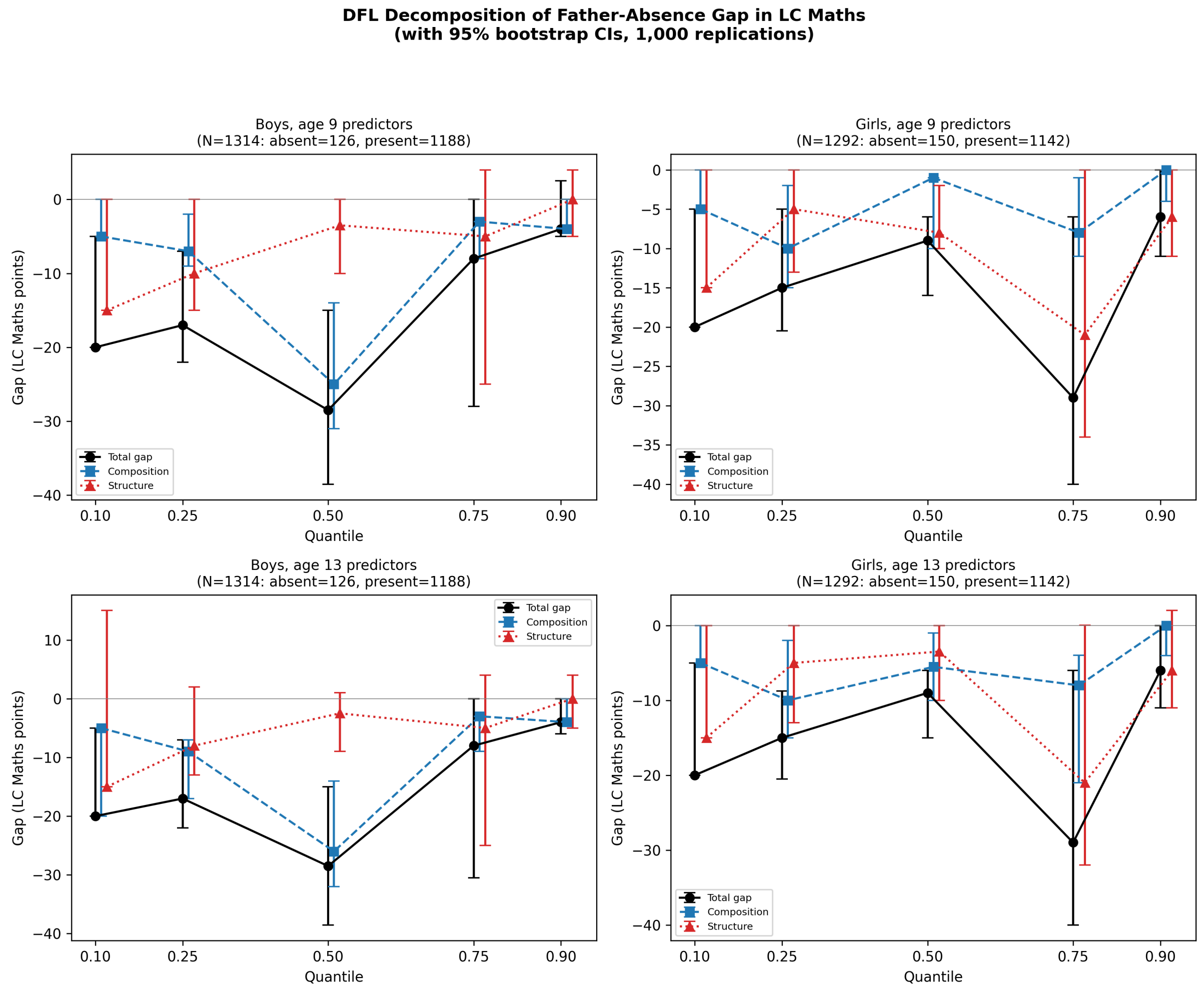

Figure 9: DFL decomposition of the father-absence gap in LC Maths across quantiles, by gender, with 95% bootstrap confidence intervals (1,000 stratified replications). Top row: age 9 predictors; bottom row: age 13 predictors; left column: boys; right column: girls. Black circles: total gap (absent − present). Blue squares: composition effect (counterfactual − present). Red triangles: structure effect (absent − counterfactual). Subtitles report cell sizes. The wider confidence intervals relative to the gender decomposition reflect the smaller father-absence subsamples ($N_{\text{absent}} \approx 126$–$150$ per gender).

Table 30 and Figure 9 show substantial father-absence penalties across the distribution. For boys (left panels), the total gap peaks at the median (−28.5 points, SE = 6.3), and composition explains 88–91% of that median gap across waves. At the lower tail, however, composition explains less, so structure effects account for more of the penalty.

For girls (right panels), the shape differs: the largest gap appears at $q_{0.75}$ (−29 points,

Table 30: The father-absence penalty in Maths is distributed broadly across quantiles rather than concentrated at the median. DFL reweighting decomposition of adjusted LC Maths points by gender; composition effects explain 88–91% for boys at the median but vary sharply for girls across waves. Bootstrap SEs from 1,000 replications.

Specification	Component	$q_{0.10}$	$q_{0.25}$	$q_{0.50}$	$q_{0.75}$	$q_{0.90}$
Boys, W1	Total gap	-20.0	-17.0	-28.5	-8.0	-4.0
		(6.9)	(4.0)	(6.3)	(6.2)	(2.2)
	Composition	-5.0	-7.0	-25.0	-3.0	-4.0
		(1.8)	(2.0)	(5.4)	(1.9)	(1.4)
	Composition %	25%	41%	88%	38%	100%
Girls, W1	Total gap	-20.0	-15.0	-9.0	-29.0	-6.0
		(5.9)	(3.9)	(2.7)	(11.6)	(4.0)
	Composition	-5.0	-10.0	-1.0	-8.0	0.0
		(2.1)	(3.2)	(2.7)	(2.7)	(0.9)
	Composition %	25%	67%	11%	28%	0%
Boys, W2	Total gap	-20.0	-17.0	-28.5	-8.0	-4.0
		(7.0)	(3.9)	(6.3)	(6.8)	(2.1)
	Composition	-5.0	-9.0	-26.0	-3.0	-4.0
		(5.3)	(2.2)	(5.5)	(2.1)	(1.6)
	Composition %	25%	53%	91%	38%	100%
Girls, W2	Total gap	-20.0	-15.0	-9.0	-29.0	-6.0
		(5.8)	(3.9)	(2.7)	(11.8)	(4.0)
	Composition	-5.0	-10.0	-5.5	-8.0	0.0
		(2.4)	(3.4)	(3.2)	(4.2)	(1.2)
	Composition %	25%	67%	61%	28%	0%

Notes: Bootstrap standard errors in parentheses from 1,000 stratified replications. Total gap = father absent − father present at each quantile. Composition = counterfactual − present (endowment differences). Structure = absent − counterfactual (coefficient differences). Father absence is defined as the secondary caregiver (father) failing to participate in both Wave 1 and Wave 2 parental surveys. Outcome: adjusted LC Maths points (0–100 scale).

SE = 11.6–11.8), and the composition share at the median is smaller, rising from 11% with age 9 predictors to 61% with age 13 predictors. Confidence bands are wider at $q_{0.75}$ because father-absent cells are small, so that quantile should be interpreted cautiously. For both genders, gaps narrow at the top of the distribution ($q_{0.90}$: 4–6 points).

Compared with the gender decomposition, the father-absence penalty is more broadly distributed across quantiles and is generally more composition-driven. This is consistent with the view that observed cognitive and family-background characteristics are more tightly linked to father-absence differences than to gender differences.

These distributional results complement the mean Oaxaca decompositions and show that father-absence associations are not confined to one part of the score distribution.

I Appendix I. DFL Distributional Decomposition of Gender Gaps in English Achievement

The English DFL decomposition uses the same reweighting procedure and 1,000 stratified bootstrap replications. Table 4 and Figure 10 show that the English gender gap (3.1 points favouring girls) is concentrated near the lower tail and is small from the median upward. With age-13 predictors, the mean composition effect turns negative (-1.49 points), meaning boys hold stronger measured endowments while girls still score higher. The coefficient component therefore explains more than 100% of the mean gap at age 13, consistent with the subject reversal in the Oaxaca results.

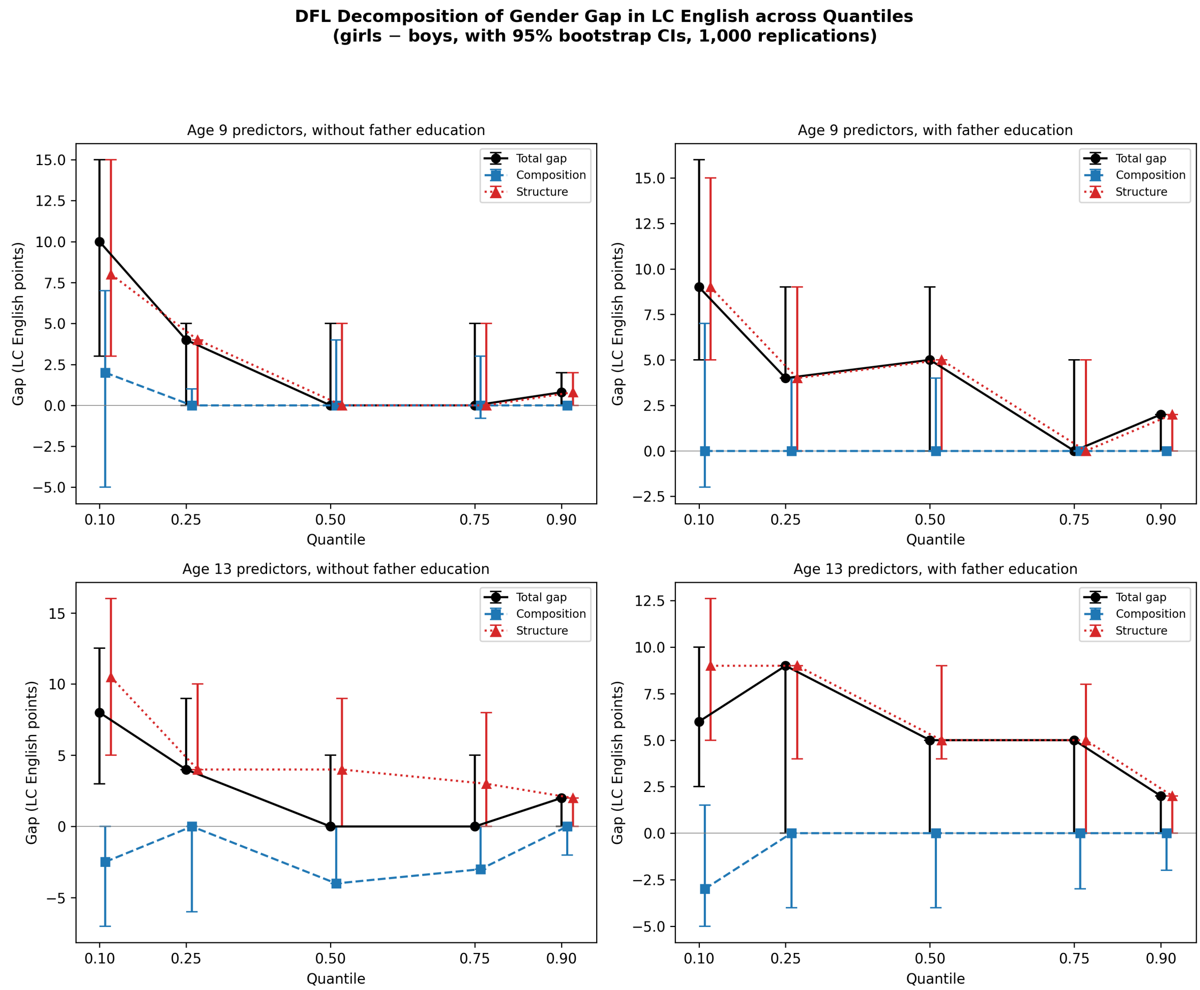

Figure 10: DFL decomposition of the gender gap in LC English across quantiles, with 95% bootstrap confidence intervals (1,000 stratified replications). Top row: age 9 predictors; bottom row: age 13 predictors; left column excludes father's education; right column includes it. The gender gap in English, which favours girls, is concentrated at the lower tail of the distribution.

J Appendix J. DFL Distributional Decomposition of Father-Absence Gaps in English Achievement

Table 31 and Figure 11 present the English father-absence decomposition. For boys, the average penalty (5.7 points) is smaller than in Maths, and composition explains about two thirds of the gap across waves. For girls, the average penalty (4.2 points) has a distinctive Wave 2 profile in which composition exceeds the total gap, implying that measured-skill differences more than account for the observed difference while the structure component offsets part of it.

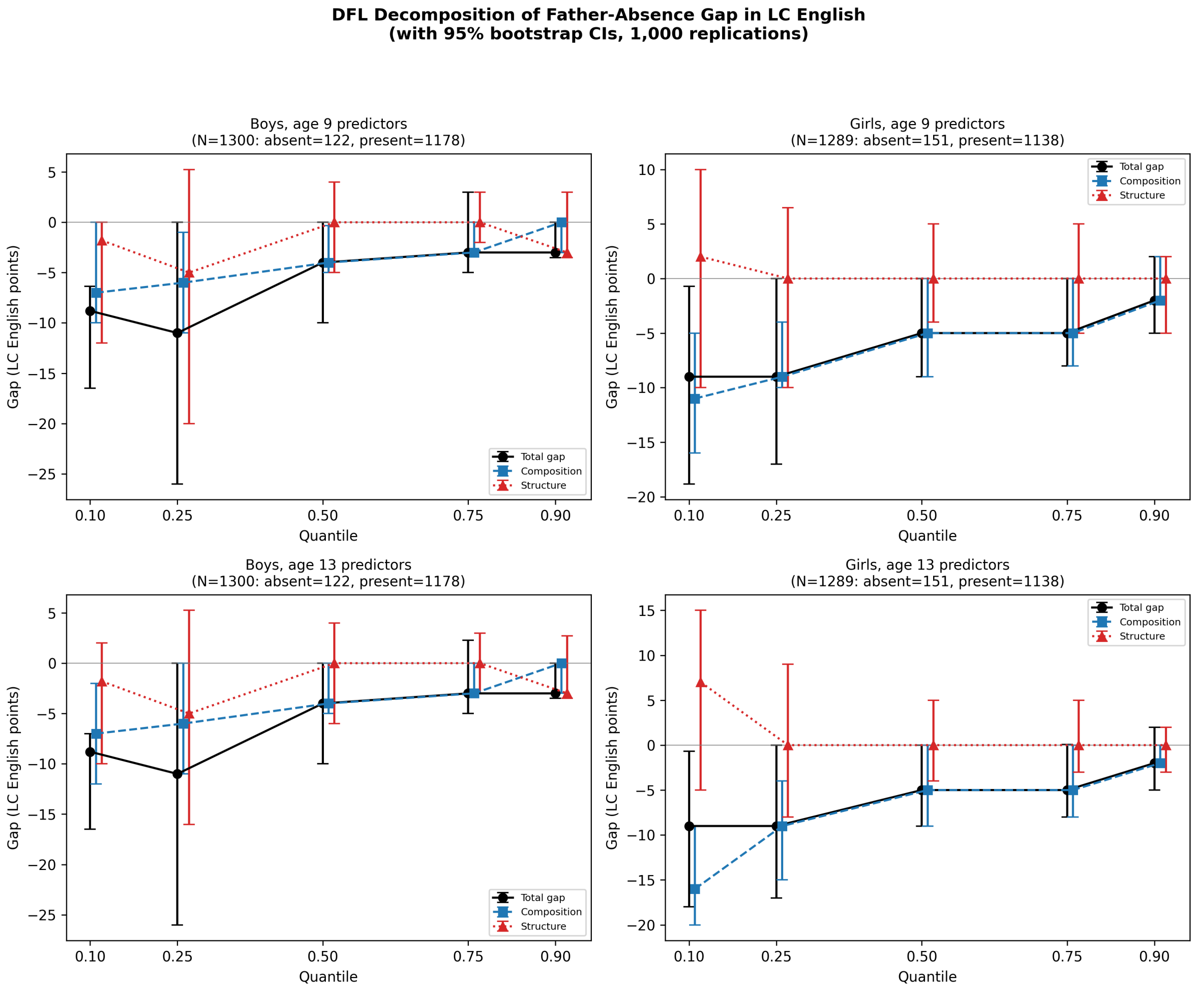

Figure 11: DFL decomposition of the father-absence gap in LC English across quantiles, by gender, with 95% bootstrap confidence intervals (1,000 stratified replications). Top row: age 9 predictors; bottom row: age 13 predictors; left column: boys; right column: girls.

Table 31: The father-absence penalty in English is smaller than in Maths; for girls with age 13 predictors, the composition effect exceeds the total gap. DFL reweighting decomposition of adjusted LC English points by gender; bootstrap SEs from 1,000 replications.

Specification	Component	$q_{0.10}$	$q_{0.25}$	$q_{0.50}$	$q_{0.75}$	$q_{0.90}$
Boys, W1	Total gap	-8.8	-11.0	-4.0	-3.0	-3.0
		(3.2)	(6.3)	(2.5)	(1.9)	(1.7)
	Composition	-7.0	-6.0	-4.0	-3.0	0.0
		(3.1)	(3.3)	(0.7)	(0.9)	(1.1)
	Composition %	80%	55%	100%	100%	-0%
Girls, W1	Total gap	-9.0	-9.0	-5.0	-5.0	-2.0
		(4.8)	(4.2)	(2.3)	(3.0)	(2.1)
	Composition	-11.0	-9.0	-5.0	-5.0	-2.0
		(3.5)	(2.6)	(1.8)	(2.5)	(1.0)
	Composition %	122%	100%	100%	100%	100%
Boys, W2	Total gap	-8.8	-11.0	-4.0	-3.0	-3.0
		(3.0)	(6.3)	(2.5)	(1.8)	(1.6)
	Composition	-7.0	-6.0	-4.0	-3.0	0.0
		(2.6)	(3.8)	(1.5)	(1.2)	(0.9)
	Composition %	80%	55%	100%	100%	-0%
Girls, W2	Total gap	-9.0	-9.0	-5.0	-5.0	-2.0
		(4.7)	(4.4)	(2.3)	(3.1)	(2.1)
	Composition	-16.0	-9.0	-5.0	-5.0	-2.0
		(2.8)	(2.7)	(2.2)	(2.4)	(0.9)
	Composition %	178%	100%	100%	100%	100%

Notes: Bootstrap standard errors in parentheses from 1,000 stratified replications. Total gap = father absent − father present at each quantile. Composition = counterfactual − present (endowment differences). Structure = absent − counterfactual (coefficient differences). Father absence defined as secondary caregiver failing to participate in both Wave 1 and Wave 2 parental surveys. Outcome: LC English points.

K Appendix K. School-Clustering Inference Sensitivity

This appendix reports a sensitivity exercise for unmodelled school clustering in the Oaxaca decompositions. Because school identifiers are unavailable in this analysis file, true cluster-robust or multilevel inference cannot be estimated directly. As a transparent second-best check, I inflate reported bootstrap standard errors by design-effect factors of 1.3 and 1.5, then re-evaluate headline significance for gender decompositions (Table 32) and father-absence decompositions (Table 33).

Table 32: Inference sensitivity to unmodelled school clustering in gender decompositions: headline Oaxaca components under design-effect standard-error inflation. Baseline columns use reported bootstrap standard errors; inflated columns multiply standard errors by 1.3 and 1.5, respectively. Significance codes use the decomposition convention ($^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$).

Spec	Component	Estimate	SE	Baseline	×1.3	×1.5
M-W1-NF	Difference	-5.212	(0.980)	***	***	***
M-W1-NF	Endowments	-1.570	(0.610)	**	**	*
M-W1-NF	Coefficients	-4.215	(0.900)	***	***	***
M-W1-WF	Difference	-4.434	(1.020)	***	***	***
M-W1-WF	Endowments	-1.298	(0.656)	**	–	–
M-W1-WF	Coefficients	-3.641	(0.924)	***	***	***
M-W2-NF	Difference	-4.882	(0.998)	***	***	***
M-W2-NF	Endowments	-4.150	(0.742)	***	***	***
M-W2-NF	Coefficients	-1.236	(0.855)	–	–	–
M-W2-WF	Difference	-4.635	(1.150)	***	***	***
M-W2-WF	Endowments	-4.039	(0.822)	***	***	***
M-W2-WF	Coefficients	-1.128	(0.963)	–	–	–
E-W1-NF	Difference	3.079	(0.649)	***	***	***
E-W1-NF	Endowments	0.260	(0.398)	–	–	–
E-W1-NF	Coefficients	2.360	(0.618)	***	***	**
E-W1-WF	Difference	3.566	(0.677)	***	***	***
E-W1-WF	Endowments	0.456	(0.393)	–	–	–
E-W1-WF	Coefficients	2.821	(0.630)	***	***	***
E-W2-NF	Difference	2.906	(0.678)	***	***	***
E-W2-NF	Endowments	-1.588	(0.465)	***	***	**
E-W2-NF	Coefficients	4.548	(0.655)	***	***	***
E-W2-WF	Difference	3.135	(0.726)	***	***	***
E-W2-WF	Endowments	-1.123	(0.486)	**	*	–
E-W2-WF	Coefficients	4.396	(0.685)	***	***	***

Note: Sensitivity exercise motivated by GUI's school-clustered sampling and the absence of school identifiers in this analysis file. This does not replace cluster-robust inference; it shows how threshold-based significance changes under plausible design-effect multipliers. Spec codes: M/E = Maths/English; W1/W2 = predictor wave; NF/WF = no/with father's education.

Table 33: Inference sensitivity to unmodelled school clustering in father-absence decompositions: headline Oaxaca components under design-effect standard-error inflation. Baseline columns use reported bootstrap standard errors; inflated columns multiply standard errors by 1.3 and 1.5, respectively. Significance codes use the decomposition convention ($^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$).

Spec	Component	Estimate	SE	Baseline	×1.3	×1.5
M-FA-W1-B	Difference	13.564	(2.921)	***	***	***
M-FA-W1-B	Endowments	5.984	(2.502)	**	*	–
M-FA-W1-B	Coefficients	6.612	(2.655)	**	*	*
M-FA-W1-G	Difference	15.225	(2.415)	***	***	***
M-FA-W1-G	Endowments	3.122	(2.156)	–	–	–
M-FA-W1-G	Coefficients	7.456	(2.270)	***	**	**
M-FA-W2-B	Difference	13.564	(2.875)	***	***	***
M-FA-W2-B	Endowments	7.406	(2.456)	***	**	**
M-FA-W2-B	Coefficients	4.992	(2.482)	**	–	–
M-FA-W2-G	Difference	15.225	(2.520)	***	***	***
M-FA-W2-G	Endowments	7.405	(2.189)	***	***	**
M-FA-W2-G	Coefficients	6.351	(1.977)	***	**	**
E-FA-W1-B	Difference	5.673	(2.105)	***	**	*
E-FA-W1-B	Endowments	3.323	(1.869)	*	–	–
E-FA-W1-B	Coefficients	1.930	(1.890)	–	–	–
E-FA-W1-G	Difference	4.179	(1.707)	**	*	–
E-FA-W1-G	Endowments	2.980	(1.554)	*	–	–
E-FA-W1-G	Coefficients	0.742	(1.572)	–	–	–
E-FA-W2-B	Difference	5.673	(2.036)	***	**	*
E-FA-W2-B	Endowments	4.112	(1.864)	**	*	–
E-FA-W2-B	Coefficients	1.975	(1.728)	–	–	–
E-FA-W2-G	Difference	4.179	(1.672)	**	*	*
E-FA-W2-G	Endowments	4.376	(1.469)	***	**	**
E-FA-W2-G	Coefficients	-0.684	(1.492)	–	–	–

Note: Sensitivity exercise motivated by GUI's school-clustered sampling and the absence of school identifiers in this analysis file. This does not replace cluster-robust inference; it shows how threshold-based significance changes under plausible design-effect multipliers. Spec codes: M/E = Maths/English; FA = father-absence decomposition; W1/W2 = predictor wave; B/G = boys/ girls.

L Appendix L. Level-Selection Sensitivity in LC Maths

Because LC Maths points pool Higher and Ordinary levels, this appendix tests how sensitive headline gender decompositions are to that mixture. Figure 12 shows the bimodal distribution and the data-driven antimode split used to define lower- and upper-support samples. I then re-estimate the four headline gender Oaxaca specifications (Wave 1/2, with/without father's education) within each support band (Table 34).

Interpretation: restricted-support results indicate that the largest pooled-score contrasts are concentrated in the mixed-support comparison, which is consistent with an important role for level composition in pooled LC points. At the same time, the central

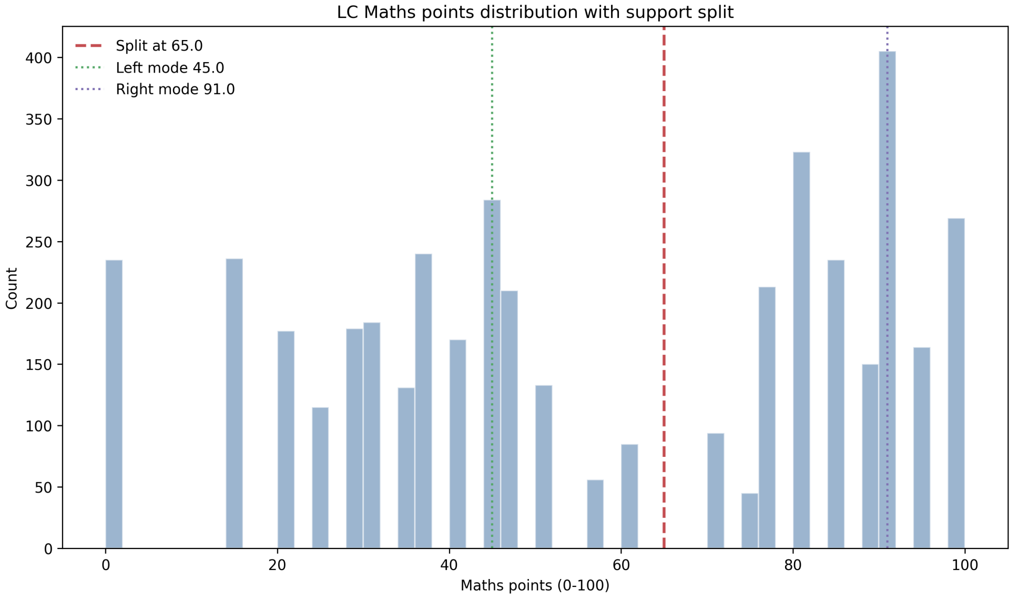

Figure 12: LC Maths points distribution with data-driven support split at the empirical antimode. The vertical dashed line marks the split used for lower- versus upper-support sensitivity samples in Appendix L. Dotted lines indicate the two modal regions.

full-sample pattern remains unchanged: Wave 1 is more coefficient-led, while Wave 2 is more endowment-led.

Table 34: Support-restriction sensitivity for the LC Maths gender decomposition. The sample is split at the antimode of the observed bimodal score distribution (Figure 12), and Oaxaca components are re-estimated in lower- and upper-support subsamples.

Specification	Sample	Difference	Endowments	Coefficients	Endowment share
W1 no father	Full sample	-5.212	-1.570	-4.215	30.1%
	Lower support	0.066	0.554	-0.327	n/a
	Upper support	-1.257	-0.135	-1.254	10.7%
W1 with father	Full sample	-4.434	-1.298	-3.641	29.3%
	Lower support	0.911	0.639	0.231	n/a
	Upper support	-1.243	-0.128	-1.279	10.3%
W2 no father	Full sample	-4.882	-4.150	-1.236	85.0%
	Lower support	0.280	-0.623	1.076	n/a
	Upper support	-1.426	-0.212	-1.241	14.9%
W2 with father	Full sample	-4.635	-4.039	-1.128	87.1%
	Lower support	0.435	-0.373	1.252	n/a
	Upper support	-1.236	-0.288	-1.146	23.3%

Note: This is a sensitivity check, not a clean level-specific identification strategy. The split is data-driven and intended to test whether the Wave 1 versus Wave 2 composition shift is mechanically driven by level-mix differences in the pooled points outcome.

M Appendix M. Attrition Robustness via Lee Bounds

This appendix reports Lee (2009) monotonic-selection bounds for the father-absence gap in LC Maths points. The treatment group is father-absent status (Waves 1 and 2 non-response), and outcomes are observed only for cases with valid Wave 4 LC reports. Because selection is lower in the father-absent group, the Lee procedure trims the selected father-present distribution to match that selection rate. Table 35 reports the bounds.

Table 35: Lee (2009) monotonic-selection bounds for the father-absence gap in LC Maths points. Treatment is father-absent status (W1+W2 non-response); outcome is LC Maths points. Bounds are for (father-absent − father-present), so negative values indicate a penalty associated with father absence.

Sample	Naive diff	Lower bound	Upper bound	Sel. absent	Sel. present	Trim share
Overall	-18.303	-30.549	-5.730	57.8%	73.1%	20.9%
Girls	-16.460	-27.857	-4.075	58.0%	73.3%	20.9%
Boys	-19.689	-32.791	-7.125	57.6%	72.9%	21.0%

Note: When selection is lower in the father-absent group, Lee bounds trim the selected father-present outcome distribution to match the father-absent selection rate under monotonic selection. This provides worst-case bounds for attrition bias from differential selection into observed outcomes.

Interpretation: bounds remain negative overall and by gender, indicating that the father-absence penalty does not disappear under monotonic-selection correction, even though magnitudes are interval-valued rather than point-identified.

N Appendix N. Additional Defence Robustness Checks

This appendix reports three targeted checks. First, a harmonised Wave 1 versus Wave 2 timing test retains only conceptually matched covariates across waves; the endowment-share shift remains strong in Maths and reverses in English. Second, the level-mix concern is split into an extensive margin (probability of upper-support placement) and an intensive margin (points conditional on upper support), which clarifies where composition effects are concentrated. Third, father-absence decompositions are re-estimated under alternative definitions using Wave 3 partner and SCG-participation information to separate structural absence from resident disengagement.

Why the timing shift is not just changing covariates. The baseline decompositions use wave-specific predictor sets because the GUI instruments change between ages 9 and 13. A natural objection is that the coefficient-to-endowment shift is an artefact of richer or better-scaled Wave 2 measurement rather than a developmental change. The harmonised specification addresses this by restricting both waves to covariates that share the same conceptual content:

- Covariates available at both waves (retained in harmonised specification): one numeracy measure, one reading/verbal reasoning measure, four SDQ subscales (emotional, conduct, hyperactivity, peer problems), primary caregiver education (two dummies), equivalised household income quintile, mixed-school indicator, father-education-missing indicator, and a grading-system dummy.
- Covariates that differ across waves: the cognitive instruments themselves change (BAS-derived scores at Wave 1; Drumcondra Numerical Ability and Verbal Reasoning at Wave 2). These are conceptually matched (numeracy and verbal/reading) but not identical tests.
- Covariates available only at Wave 2 (excluded from harmonised specification): fee-paying school status, DEIS status, and secondary caregiver education.

Table 36 shows the result. With these harmonised covariate sets, the endowment-share shift is 51 percentage points in Maths (no father-education control) and 62 percentage points (with father-education control), both significant at $p < 0.001$. In English, the shift reverses direction by 35–53 percentage points. The timing pattern therefore does not depend on Wave 2-specific school controls or the availability of additional predictor variables; it survives when the covariate structure is held as constant as the data allow.

Table 36: Wave-comparability check using harmonised covariate sets across Waves 1 and 2.

Specification	W1 endow. (%)	W2 endow. (%)	Shift	SE	p-val.	Diff. endow.	SE	p-val.
Maths no father (harmonised)	38.98	89.94	50.96	16.44	0.000	-2.79	0.65	0.000
Maths with father (harmonised)	39.64	101.61	61.97	31.37	0.000	-2.75	0.73	0.000
English no father (harmonised)	-1.03	-54.38	-53.35	21.51	0.000	-1.69	0.41	0.000
English with father (harmonised)	6.82	-29.03	-35.86	15.03	0.000	-1.43	0.45	0.000

Table 37: Two-part level-mix analysis using an antimode split at 65.0: extensive-margin decomposition of upper-support probability and intensive-margin decomposition of points within upper support.

Specification	Margin	N	Difference	Endowments	Coefficients	Endow. share (%)
W1 No father	Extensive: P(upper support)	3690	-0.086	-0.034	-0.063	39.804
W2 No father	Extensive: P(upper support)	3617	-0.078	-0.070	-0.021	89.101
W1 No father	Intensive: points | upper support	1581	-1.257	0.018	-1.342	-1.438
W2 No father	Intensive: points | upper support	1576	-1.380	-0.252	-1.265	18.295
W1 With father	Extensive: P(upper support)	3241	-0.081	-0.029	-0.059	36.440
W2 With father	Extensive: P(upper support)	2940	-0.075	-0.067	-0.020	89.985
W1 With father	Intensive: points | upper support	1454	-1.243	0.011	-1.358	-0.917
W2 With father	Intensive: points | upper support	1363	-1.254	-0.256	-1.220	20.384

Table 38: Father-absence robustness grid using alternative definitions available in the data (baseline non-response, structural absence, resident disengagement).

Definition	Wave	Group	N	Difference	Endowments	Coefficients	Endow. share (%)
Baseline W1+W2 non-response	W1	All	3153	14.433	5.182	6.322	35.906
Baseline W1+W2 non-response	W1	Boys	1570	13.172	5.637	6.017	42.795
Baseline W1+W2 non-response	W1	Girls	1583	14.876	3.750	6.482	25.207
Baseline W1+W2 non-response	W2	All	3164	15.333	8.502	5.882	55.446
Baseline W1+W2 non-response	W2	Boys	1564	14.685	6.894	5.714	46.948
Baseline W1+W2 non-response	W2	Girls	1600	15.412	8.314	6.077	53.942
Structural (no resident partner, W3)	W1	All	2532	13.633	6.082	7.085	44.615
Structural (no resident partner, W3)	W1	Boys	1264	12.535	5.604	6.860	44.709
Structural (no resident partner, W3)	W1	Girls	1268	13.830	4.695	7.252	33.948
Structural (no resident partner, W3)	W2	All	2652	13.450	7.741	5.012	57.558
Structural (no resident partner, W3)	W2	Boys	1314	12.640	7.641	5.336	60.455
Structural (no resident partner, W3)	W2	Girls	1338	13.467	6.687	4.944	49.656
Resident disengaged (partner present, no SCG W3)	W1	All	2634	5.444	3.025	2.249	55.571
Resident disengaged (partner present, no SCG W3)	W1	Boys	1326	5.479	2.143	2.851	39.114
Resident disengaged (partner present, no SCG W3)	W1	Girls	1308	5.032	4.038	1.466	80.246
Resident disengaged (partner present, no SCG W3)	W2	All	2756	5.230	4.086	0.998	78.123
Resident disengaged (partner present, no SCG W3)	W2	Boys	1376	5.547	2.620	2.654	47.223
Resident disengaged (partner present, no SCG W3)	W2	Girls	1380	4.586	5.549	-0.377	121.011

O Appendix O. Attrition Balance by Gender

Table 39 compares Wave 1 baseline covariates for children retained in the analytical sample ($N = 4{,}213$) versus those lost to attrition ($N = 1{,}681$), separately by gender and pooled. Standardised differences exceeding $|0.10|$ indicate non-trivial imbalance; values exceeding $|0.25|$ are conventionally large.

Retained children score 0.3–0.4 SD higher on Wave 1 cognitive tests and come from households with higher income and more educated primary caregivers. They also display fewer externalising behaviours (lower SDQ conduct and hyperactivity scores). The pattern is broadly parallel for boys and girls, suggesting that attrition is positively selected on advantage but not strongly differential by gender. These imbalances motivate the IPW reweighting and Lee bounding exercises reported in Appendices K and M.

Table 39: Attrition balance: retained vs. lost observations on Wave 1 covariates, by gender.

Variable	Mean (Retained)	Mean (Lost)	Std. Diff.
Panel: Boys (Retained $N = 2,050$; Lost $N = 820$)			
Cognitive Maths (W1, log)	-0.377	-0.707	+0.355
Cognitive Reading (W1, log)	0.353	-0.069	+0.431
SDQ Emotional (W1)	1.811	1.908	-0.052
SDQ Conduct (W1)	1.190	1.460	-0.189
SDQ Hyperactivity (W1)	3.025	3.587	-0.228
SDQ Peer Problems (W1)	1.068	1.202	-0.092
PCG Educ: Secondary (W1)	0.557	0.571	-0.028
PCG Educ: Degree+ (W1)	0.336	0.244	+0.200
Income Quintile (W1)	3.553	3.239	+0.237
Panel: Girls (Retained $N = 2,163$; Lost $N = 861$)			
Cognitive Maths (W1, log)	-0.556	-0.830	+0.319
Cognitive Reading (W1, log)	0.355	0.021	+0.357
SDQ Emotional (W1)	2.049	2.252	-0.102
SDQ Conduct (W1)	1.037	1.345	-0.231
SDQ Hyperactivity (W1)	2.421	3.030	-0.273
SDQ Peer Problems (W1)	1.071	1.167	-0.069
PCG Educ: Secondary (W1)	0.561	0.585	-0.050
PCG Educ: Degree+ (W1)	0.310	0.193	+0.264
Income Quintile (W1)	3.463	3.186	+0.207
Panel: Pooled (Retained $N = 4,213$; Lost $N = 1,681$)			
Cognitive Maths (W1, log)	-0.469	-0.770	+0.336
Cognitive Reading (W1, log)	0.354	-0.023	+0.394
SDQ Emotional (W1)	1.933	2.085	-0.078
SDQ Conduct (W1)	1.111	1.401	-0.209
SDQ Hyperactivity (W1)	2.715	3.302	-0.248
SDQ Peer Problems (W1)	1.069	1.184	-0.081
PCG Educ: Secondary (W1)	0.559	0.578	-0.039
PCG Educ: Degree+ (W1)	0.323	0.218	+0.232
Income Quintile (W1)	3.507	3.212	+0.221

Notes: Baseline is the broader pre-analytical sample ($N = 5,894$). Retained observations appear in the decomposition analytical sample ($N = 4,333$). Standardised difference $= (\bar{x}_{\text{ret}} - \bar{x}_{\text{lost}})/s_{\text{pooled}}$. Values exceeding $|0.10|$ suggest non-trivial imbalance; exceeding $|0.25|$ is conventionally large. Variables with no lost-group coverage are omitted from this table but reported in the companion CSV.

P Appendix P. LC Maths Reporting Diagnostics

A concern with self-reported examination outcomes is that respondents may round, heap, or misreport their scores. Figure 13 plots the distribution of LC Maths points for boys and girls in the analytical sample.

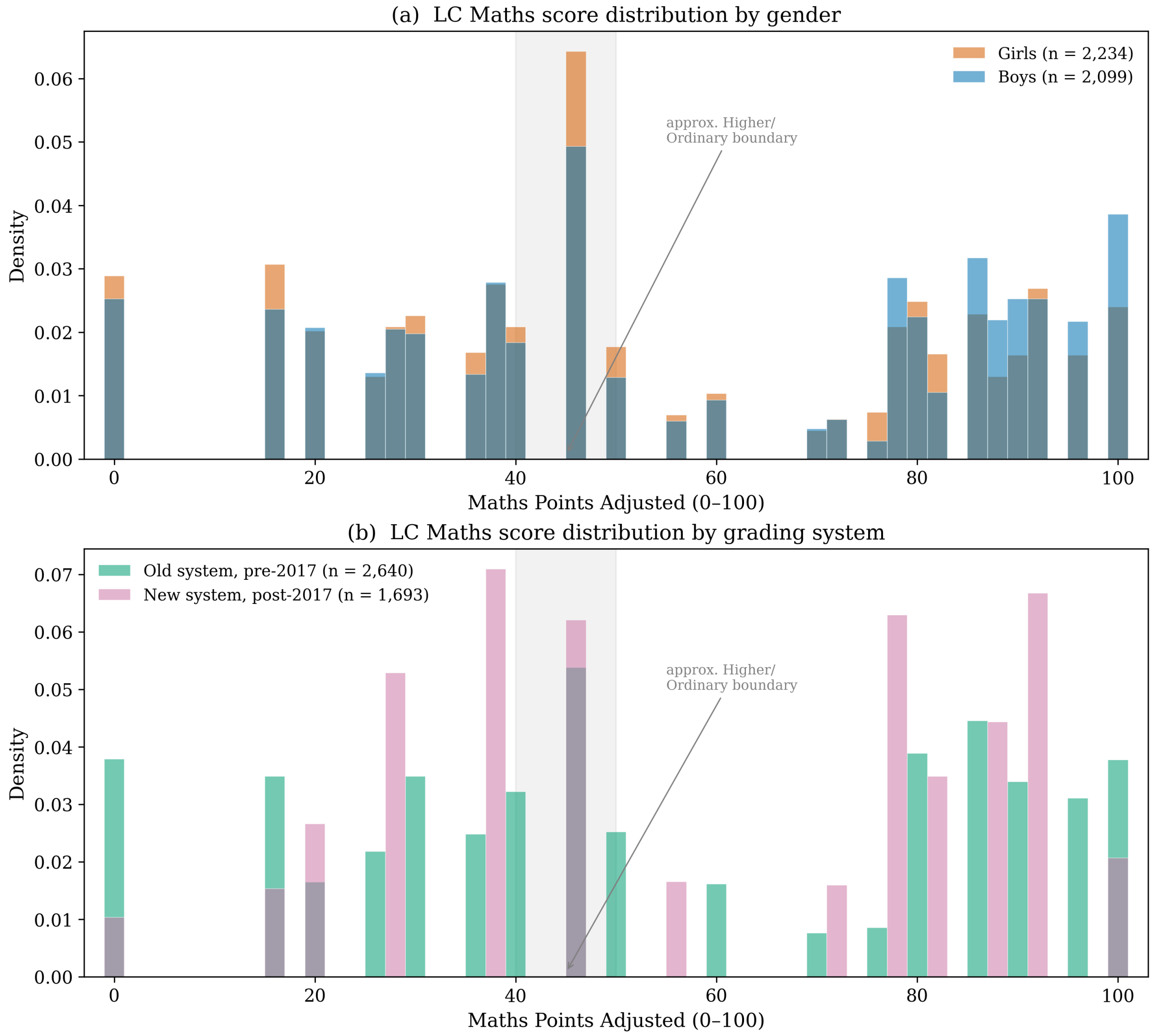

Figure 13: Distribution of self-reported LC Maths points by gender ($N = 4{,}213$). The distribution exhibits 26 discrete values corresponding to grade boundaries under the Higher and Ordinary Level grading systems, producing a clear bimodal structure. The absence of inter-boundary heaping suggests that respondents report actual grades rather than approximate point totals.

The distribution shows 26 discrete values aligned with official grade boundaries, not a continuous or heaped distribution. The bimodal structure reflects the Higher/Ordinary Level split, with boys shifted slightly rightward. The discretisation implies that respondents are reporting actual letter grades (which map mechanically to points) rather than free-form point estimates, limiting the scope for idiosyncratic misreporting. This does not rule out systematic level-selection bias (addressed in Appendix L), but it does indicate that the outcome variable is grade-boundary determined rather than self-estimated.